ANSWERS

1.01

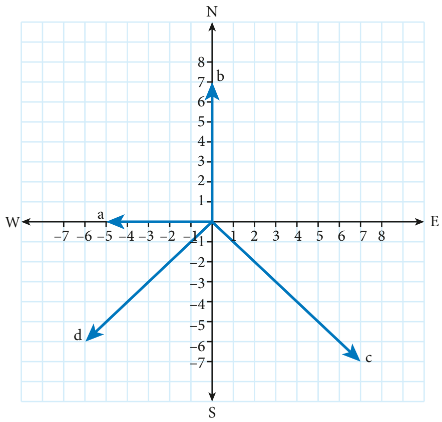
1 B

2 E

3 E

4 B

5



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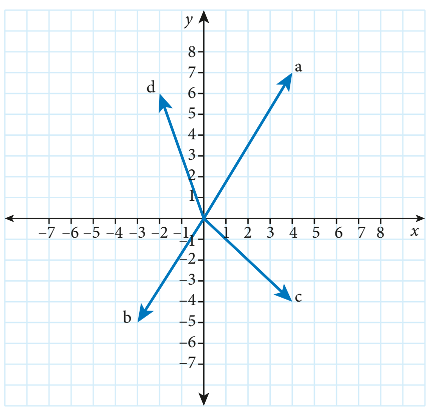

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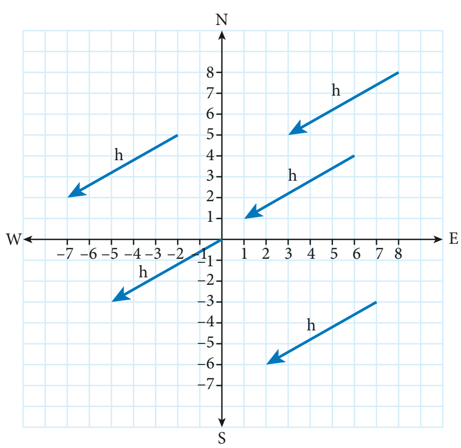


8 a 5 b 13 c 25 d $\sqrt{2}$

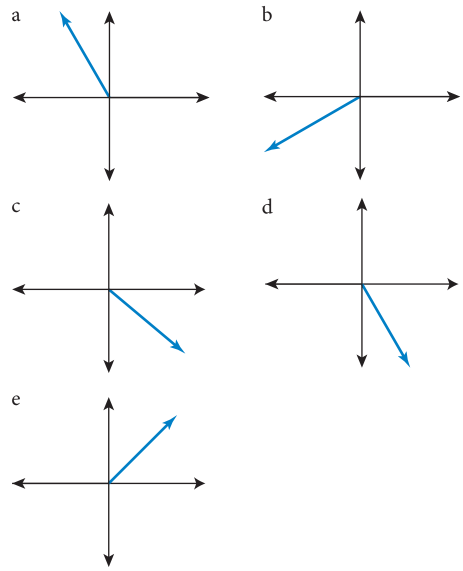
e $\sqrt{53}$ f $3\sqrt{5}$ g $\sqrt{a^2+16}$ h $\sqrt{36+b^2}$

9 a 53.1° b 112.6°

c 343.7° d 225°

e 105.9° f 296.6°

10 The vectors in your answer should be in the same direction but the scale could be different.



11 a (5, 306.9°) b (7.07, 135°)

c (12.21, 235.0°) d (12.65, 71.6°)

e (18.87, 122.0°)

12 a (17, 61.9°) b (13, 22.6°)

c (15, 233.1°) d (13.9, 239.7°)

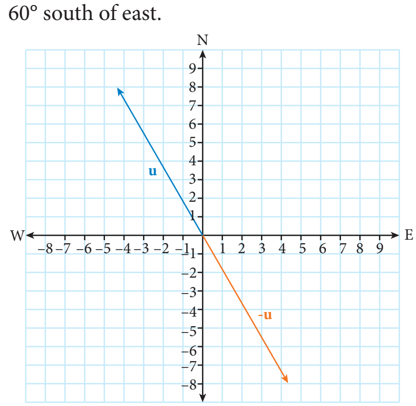
e (11.3, 315°)

13 a $AB \approx (7.3, 74.1°)$ b $RB \approx (5.8, 211.0°)$

c $MB \approx (10.8, 123.7°)$

14 a $-\mathbf{u}$ has a magnitude of 8 and is in the direction 60° south of east.

b



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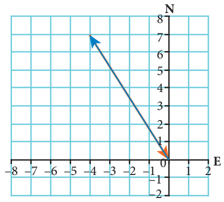
c



d The total displacement is 0.

1.02

1 D

2 B

3 a 14 east b 57 east
c 23 east d 7 west
e 73 east

4 a 60 N up b 9 N down
c 26 N up d 103 N down
e 110 N east

5 a (7.21, 146.1°) b (4.50, 12.6°)
c (10.93, 9.3°) d (10.82, 276.3°)
e (16.93, 77.7°)

6 a (13, 67.8°) b (6.4, 292.7°)
c (6.1, 274.7°) d (20.4, 220.3°)
e (24.1, 265.4°)

7 About 16.8 km on a bearing of 075.4° (14.6° north of east).

8 About 668 N at 14.9° to the vertical.

9 About 1433.2 m on a bearing of 347.6° (77.6° north of west).

10 About 9.6 km on a bearing of 089.8°.

11 About 37.1 km on a bearing of 062.8° (27.2° north of east).

12 22.4 ms^{-1} on a bearing of 167.2° (12.8° east of south).

1.03

1 a 74.1° b 83.7° c 63.4° d 58.0°
e 102.5° f 146.3° g 221.2° h 337.4°

2 a 5 b 13 c 6.32 d 11.70
e 11.40 f 15.26 g 13.89 h 13.15

3 a (5, 306.9°) b (7.1, 135°)
c (12.2, 235.0°) d (12.7, 71.6°)
e (18.9, 122.0°) f (17, 61.9°)
g (13, 22.6°) h (15, 233.1°)
i (13.9, 210.3°) j (11.3, 315°)

4 a (15, 53.13°) b (20.25, 57.09°)
c (17.49, 120.96°) d (22.20, 324.16°)
e (26.25, 220.37°) f (18.56, 27.95°)
g (14.30, 296.46°) h (12.26, 108.60°)

5 a (4.33, 2.5) b (5, −8.66)
c (0, 24) d (−11.31, 11.31)
e (−14, −24.25) f (0, −70)
g (35, 0) h (−22, 0)

6 a (3, 5.20) b (−6, 10.39)
c (10.47, 13.40) d (−22.32, 5.56)
e (−9.37, −10.40) f (4.39, −14.34)
g (−18.11, −15.75) h (41.12, −12.57)
i (−3.76, −1.37) j (4, −6.93)

7 a (1, 5) b (3, 9)
c (3, 8) d (7, 7)
e (−3, −10) f (6, −11)

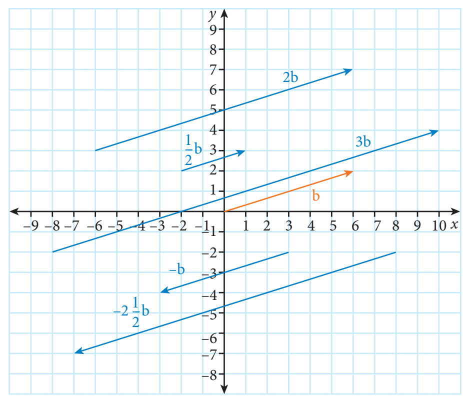
8 (10.63, 311.2°)

1.04

1 B

2 B

3



4 a $\begin{bmatrix} 54 \\ -36 \end{bmatrix}$ b $\begin{bmatrix} -36 \\ 24 \end{bmatrix}$
c $\begin{bmatrix} 6 \\ -4 \end{bmatrix}$ d $\begin{bmatrix} 115.2 \\ -76.8 \end{bmatrix}$
e $\begin{bmatrix} 9 \\ -6 \end{bmatrix}$ f $\begin{bmatrix} 45 \\ -30 \end{bmatrix}$
g $\begin{bmatrix} 12 \\ -8 \end{bmatrix}$ h $\begin{bmatrix} -72 \\ 48 \end{bmatrix}$

5 a (18, 47°) b (24, 227°)
c (15, 47°) d (6, 227°)
e (30, 227°) f (60, 47°)
g (42, 227°) h (3, 47°)

6 a (8, 28) b (−9, 36)
c (−14, 31.5) d (19.4, 67.9)
e $(\frac{4}{3}, -3)$ f $(1\frac{1}{2}, -6)$
g $(-3, 6\frac{3}{4})$ h (−4, −14)

7 a (12, 195°) b (4, 124°)
c (7.5, 69°) d (16, 15°)
e (2.5, 249°) f $(\frac{2}{3}, 304°)$
g (20, 249°) h (20, 15°)

1.05

1 B

2 B

3 a $\left(\frac{2\sqrt{13}}{13}, \frac{3\sqrt{13}}{13}\right)$ b $\left(\frac{4\sqrt{41}}{41}, \frac{-5\sqrt{41}}{41}\right)$

c $\left(\frac{-\sqrt{10}}{10}, \frac{-3\sqrt{10}}{10}\right)$ d (0.411, −0.912)

e $\begin{bmatrix} \frac{-3\sqrt{10}}{10} \\ \frac{\sqrt{10}}{10} \end{bmatrix}$ f $\begin{bmatrix} \frac{-5}{13} \\ \frac{-12}{13} \end{bmatrix}$

g $\begin{bmatrix} \frac{24}{25} \\ \frac{-7}{25} \end{bmatrix}$ h $\begin{bmatrix} 0.869 \\ 0.496 \end{bmatrix}$

i (1, 86°) j (1, −165°)

k $\left(1, \frac{2\pi}{3}\right)$ l $\left(1, \frac{7\pi}{5}\right)$

4 a $5\mathbf{i} - 3\mathbf{j}$ b $-6\mathbf{i} + 4\mathbf{j}$ c $-4\mathbf{i} - 7\mathbf{j}$

d $6\mathbf{i} + 5\mathbf{j}$ e $-3.2\mathbf{i} - 9.4\mathbf{j}$ f $\mathbf{i} - 4\mathbf{j}$

g $-2\mathbf{i} - 8\mathbf{j}$ h $-3\mathbf{i} + 5\mathbf{j}$ i $7.59\mathbf{i} + 3.68\mathbf{j}$

j $0.07\mathbf{i} + 0.19\mathbf{j}$ k $1.93\mathbf{i} - 6.73\mathbf{j}$ l $-5.16\mathbf{i} + 7.37\mathbf{j}$

m $9.53\mathbf{i} - 5.5\mathbf{j}$ n $-8\mathbf{i} - 13.86\mathbf{j}$ o $-2.33\mathbf{i} - 4.20\mathbf{j}$

5 $\hat{\mathbf{m}} = \left(-\frac{3}{5}, \frac{4}{5}\right)$

6 $\left(\frac{24\sqrt{65}}{65}, -\frac{42\sqrt{65}}{65}\right)$

1.06

1 a (−13, 13) b (18, −14) c (−5, −1)

d (10, −11.8) e (4.17, −4.22) f $\left(1\frac{3}{4}, 2\frac{13}{15}\right)$

2 a $\begin{bmatrix} 23 \\ -17 \end{bmatrix}$ b $\begin{bmatrix} -12 \\ -2 \end{bmatrix}$ c $\begin{bmatrix} 6 \\ 8 \end{bmatrix}$ d $\begin{bmatrix} 3.3 \\ -13.3 \end{bmatrix}$

e $\begin{bmatrix} 1.1 \\ -6.44 \end{bmatrix}$ f $\begin{bmatrix} 6\frac{1}{4} \\ -5\frac{7}{9} \end{bmatrix}$

3 a $7\mathbf{i} - 13\mathbf{j}$ b $-17\mathbf{i} + 10\mathbf{j}$

c $-5\mathbf{i} + 5\mathbf{j}$ d $-3.4\mathbf{i} - 12\mathbf{j}$

e $-8.23\mathbf{i} + 5.15\mathbf{j}$ f $-10\frac{1}{10}\mathbf{i} - 9\frac{5}{8}\mathbf{j}$

4 a (−4, 5) b (0, 10)

c (5, 20) d (12, −2)

e (−9, 14) f (−30, 24)

g (5, −1) h (0, 0) or **0**

i (−34, 69) j (−4, −32)

5 a (14.33, 41.1°) b (38.65, 104.6°)

c (9.32, 324.9°) d (173.28, 271.7°)

e (6, 130°)

6 a **AB** = (2, 7) b **RB** = (−5, −3)

c **MB** = (−6, 9) d **QP** = (0, −18)

e **BX** = (8, 14) f **FG** = (8, −11)

7 $|c\mathbf{a}| = \sqrt{(cx_1)^2 + (cy_1)^2}$

$= \sqrt{c^2(x_1^2 + y_1^2)}$

$= \sqrt{c^2}\sqrt{x_1^2 + y_1^2}$

$= -c|\mathbf{a}|$ (to make it positive).

8 $c(\mathbf{a} + \mathbf{b}) = c(x_1 + x_2, y_1 + y_2)$

$= (cx_1 + cx_2, cy_1 + cy_2)$

$= (cx_1, cy_1 + cx_2, + cy_2)$

$= c(x_1, y_1) + c(x_2, y_2)$

$= c\mathbf{a} + c\mathbf{b}$

1.07

1 D

2 B

3 a $-\mathbf{m} = (5, -8)$ b $-\mathbf{d} = (11, 4)$

c $-\mathbf{r} = (0, 6)$ d $-\mathbf{q} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

e $-\mathbf{g} = \begin{bmatrix} -14 \\ -3 \end{bmatrix}$ f $-\mathbf{h} = \begin{bmatrix} 12 \\ -11 \end{bmatrix}$

g $-\mathbf{a} = -4\mathbf{i} + 9\mathbf{j}$ h $-\mathbf{p} = -\frac{2}{3}\mathbf{i} - \frac{4}{5}\mathbf{j}$

i $-\mathbf{n} = 0.24\mathbf{i} + 1.06\mathbf{j}$

4–6 Demonstrations using procedure similar to Example **15**.

7–9 Proofs using procedure similar to Example **16**.

1.08

1 About 12 340 N at 11.6° to the passenger's side forward direction.

2 6.262 km at 153.2°

3 282.8 N

4 About 696 N at an angle of 46.6° to the shoreline.

5 About 26.9 m/s at 042.0°.

6 84.85 km h^{-1} at 045°.

7 About 181.8 knots at 066.5°.

8 About 10.7 m/s at 082.5°.

9 $v\sqrt{2[1-\cos(\theta)]}$

CHAPTER 1 REVIEW

1 D

2 A

3 B

4 D

5 B

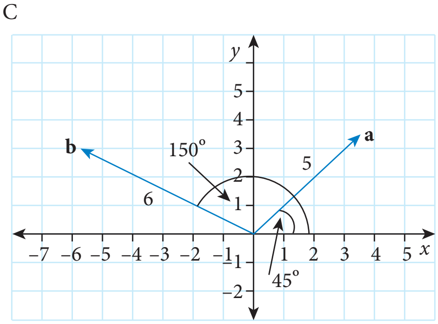
6 A

7 D

8 A

9 C

10



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11 a DE = −2i + 6j
b TS = 8i + 5j
c GJ = −4i + 4j
d RK = −2i + 4j
e YZ = 7i − 12j

12 a (9, −4) b (9, −4) c (0, 0) or **0**
d (3, 10) e (15, −1)

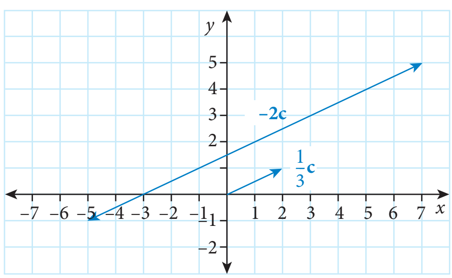
13 a (9.434, 58°) b (5, 126.9°)
c (11.662, 239°) d (9.487, 341.6°)
e (4, 180°)

14 a $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$ b $(-4\sqrt{3}, 4)$
c $(2, -2\sqrt{3})$ d $\left(\frac{7\sqrt{2}}{2}, \frac{7\sqrt{2}}{2}\right)$
e (−5.785, −6.894)

15 a (5.831, 59°) b (6.325, 251.6°)
c (4.123, 284°) d (5, 90°)
e (8.246, 104°)

16 a 26 b 5 c 17
d $\sqrt{106} \approx 10.296$ e $\sqrt{244} \approx 15.621$

17



18 $\begin{bmatrix} -72 \\ 54 \end{bmatrix}$

19 $\left(\frac{7}{25}, -\frac{24}{25}\right)$

20 a **i** − 3**j** b 4**i** + 5**j**
c −2**i** + 5**j** d 4**i** − 3**j**
e −2**i** − 5**j** f 3.36**i** + 4.97**j**
g −7.52**i** − 2.74**j** h −3**i**
i −7.88**i** + 6.16**j** j $-2.5\mathbf{i} + \frac{5\sqrt{3}}{2}\mathbf{j}$

21 a 12**i** − 5**j** b (−6, −2) c $\begin{bmatrix} 5 \\ 4 \end{bmatrix}$

22 a (2, 9) b (−5, −8) c (−8, −1)
d (21, 34) e (0, 6)

23 a $\begin{bmatrix} -6 \\ 4 \end{bmatrix}$ b $\begin{bmatrix} 8 \\ 3 \end{bmatrix}$ c $\begin{bmatrix} 7 \\ 8 \end{bmatrix}$
d $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$ e $\begin{bmatrix} -1 \\ 6 \end{bmatrix}$

24 a (10.39, 121.1°) b (10.92, 176.1°)
c (4.99, 158.8°) d (9.85, 55.9°)
e (7.13, 275.8°)

25 a (9.43, 208°) b (16.86, 52.1°)
c (2.43, 166.6°) d (9.43, 92°)
e (7.36, 155.4°)

26 Demonstration.

27 Proof.

28 a −**a** b −**a** + **c** c **a** + **c**
d $\frac{1}{2}(\mathbf{a} + \mathbf{c})$ e $\frac{1}{2}(-\mathbf{a} + \mathbf{c})$ f $\frac{1}{2}(-\mathbf{a} + \mathbf{c})$

29 Approximately 41 N at 10.7° to the horizontal.

30 1082 m on bearing 146.3°.

31 About 53 600 N in the direction N 14.5° W.

32 0.4013 m/s at 151.2° to the original direction.

2.01

1 a inductive b deductive
c deductive d by contradiction

2 *PQRS* has an interior angle sum of 360°.

3 Sea temperatures will always rise.

4 a necessary b necessary
c necessary d if and only if

5 a False, other plants may be prickly as well.
b False, the first premise is not correct.

6 Inductive, but the conclusion is wrong, Mary may have a girl.

7 This number is divisible by 4. ∴ It is composite.

8 Other shapes may have equal diagonals as well.
Corrected: This shape is a square.
∴ it has 2 equal diagonals.

9 It may be a coincidence that they have blond hair and scored well. Maybe all students in this class decided to dye their hair blond for a charity event.

10 Mandy may have more understanding than Dennis or may have done her homework through the term. Dennis may have had a bad day or lost his calculator.

2.02

1 a $x = -3$
b Not true for $x = 3$
c In 3D space, skew lines are not parallel and do not meet.
d octopus, some crustaceans

2 a false, rectangle b false, $y = -2$
c false, $a = -1$, $b = 2$ d false $n = 0$
e false – could be similar

3 a Yes, the statement is false for $x = 4$.
b No, the statement is about where fish live so it is not a counterexample.

4 False, try $a = 0$.

5 No, only if n is positive.

6 No, in 3D a vertical edge of a cube and an edge through one of the other vertical edges are neither parallel nor intersecting.

7 No, the intersections for an obtuse-angled triangle are outside.

8 No, just make a quadrilateral by choosing a fourth point that is not on the circumscribing circle of a triangle.

9 Yes, because it is necessarily convex.

10 No, draw a concave quadrilateral.

2.03

1 a If you surf at the beach, then you live in Queensland.
  b If you like Maths, then you are an interesting person.
  c If you went to school in Darwin, then you are the Premier.
  d If an animal has big teeth, then it is a lion.
2 a If you sweat, then you do a lot of exercise. False.
  b If your Maths improves, then you study hard. False.
  c If a boy has learnt to drive, then he has a P-plate. False.
  d If an animal lives in a tree, then it is a koala. False.
3 a If $3x = 15$, then $x = 5$. $x = 5$ iff $3x = 15$.
  b If a quadrilateral is a parallelogram then it has two pairs of opposite sides equal.
  A quadrilateral has two pairs of opposite sides equal iff it is a parallelogram.
  c If $a^2 > b^2$, then $a > b$. False. Try $a = -3$.
  d If you have big shoes, then you have big feet. False. (you may be a clown)
4 Converse: If students get a better job, then they study more maths. Bob has negated the statement.
5 In a way, they were both right. Female is the opposite gender to male, but the converse of male is either female or neuter. Bees are divided into queens (female), drones (male) and workers (sexless).

2.04

1 a If you don't live underground, then you don't live in Coober Pedy.
  b If you don't have a licence, then you cannot drive a car.
  c If you don't need hearing aids, then you are not old.
  d If $x^2 \neq 4$, then $x \neq 2$.
  e If an animal is not an amphibian, then it is not a cane toad.
  f If you are not mortal, then you are not a man.
2 a If it is sunny, then the sky is blue.
  b If a student passes his exams, then he studies.
  c If you live in the desert, then you live in Alice Springs.
  d If a number is a counting number, then it is positive.
  e If a vehicle has four wheels, then it is a car.
  f If you have money, then you have a job.
3 a If $x^2 \ngeq 9$, then $x \ngeq 3$.
  b If a quadrilateral does not have diagonals equal in length, then it is not a rectangle.
  c If people do not live in Australia, then they do not live in Tasmania.
  d If an animal is not a marsupial, then it is not a koala.
4 a If $x^2 \ngeq 9$, then $x \ngeq -3$. False.
  b If an animal can't fly, then it isn't a bird. False.
  c If you will not get a good job, then you have not been to university. False.
  d If a number is not real, then it is not rational. True.
5 a If they are not similar shapes, then two triangles do not have matching equal angles. True.
  b If a quadrilateral is not a rhombus, then it does not have diagonals that meet at right angles. False.
  c If an animal does not have a trunk, then it is not an elephant. True.
6 The contrapositive is: If a quadrilateral is not a square, then it does not have four equal sides. False.
7 D

2.05

1 a $a = 35$ b $x = 22\frac{1}{2}$ c $x = 10$
  d $x = 140$ e $w = 96$ f $y = 81$
2 a Proof b Proof
3 Proof
4 a AAS b SSS c SAS d RHS
5 Proof
6 a Proof, SAS b Proof
7 Proof
8 $x = 72$

2.06

1 Proof
2 a If it is a rhombus, then the diagonals of a parallelogram meet at right angles.
  b Proof
  c Proof
  d The diagonals of a rhombus bisect each other at right angles.
3 a $(a + b, a - b)$
  b $(a - b, -b - a)$
  c The vectors are the same length as they only differ in sign or direction.
4 Proof
5 Proof
6 Proof
7 a If a line is parallel to one side of a triangle and half its length, then it bisects the other two sides.
  b Proof
  c Proof
8, 9 Proof
10 a,b Proof
  c Both pairs of opposite sides parallel (or equal).

2.07

1 a $\forall x \in \mathbf{R}, x^2 \geq 0$
  b $\exists w \in \mathbf{Q}$ such that $2 < w \leq 7$
  c $\forall$ adult human beings $\exists$ a perfect match
  d $\forall a, b \in \mathbf{R}, \exists c \in \mathbf{R}$ such that $a < c < b$
  e $\forall x \in \mathbf{R}$ such that $x \geq 0, \exists y \in \mathbf{R}$ where $y \geq 0$ such that $y = \sqrt{x}$.
2 a Proof b Proof
3 Proof

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4 Proof
5 Proof
6 Proof

CHAPTER 2 REVIEW

1 D
2 B
3 D
4 A
5 C
6 a True b False, let $x = -3$.
c True d False, the number 2 is prime.
7 Elsie improves her playing.
8 There will be floods every 2 years in Queensland.
9 a If you tell the truth, then you are a politician.
b If the square of a number is positive, then the number is positive.
c If a student must resit an exam, then he/she has failed the exam.
d If an animal can swim, then it is a fish.
10 a If you do not tell the truth, then you are not a politician
b If the square of a number is not positive, then the number is not positive.
c If a student does not have to resit an exam, then he/she has not failed the exam.
d If an animal cannot swim, then it is not a fish.
11 a No
b A number is composite iff it has more than two factors.
c A quadrilateral is a rhombus iff it has diagonals that bisect each other at right angles.
d No
12 a Contrapositive: If a quadrilateral does not have two pairs of adjacent sides equal, then it is not a kite. Yes, the statement is true.
b Contrapositive: If $a^2 \not> b^2$, then $a \not> b$. No.
c Contrapositive: If a triangle does not have two equal sides, then it does not have two equal angles. Yes, the statement is true.
d Contrapositive: If a vehicle does not have wheels, then it is not a bike. Yes, the statement is true.
13 a Proof, let $2x + 2y = 180$
b Proof, AAS c Proof, AA
14 a $\mathbf{d} = \mathbf{a} + \mathbf{c} - \mathbf{b}$ b $\mathbf{m} = \frac{1}{2}(\mathbf{a} + \mathbf{c})$
15 Proof
16 swimming $\Rightarrow$ wet, shower $\Rightarrow$ wet. There is more than one way of getting wet.
17 'if and only if', converse is also true.
18 Try $x = -\frac{1}{3}$.
19 If a triangle is isosceles, then a median of the triangle is perpendicular to the side it intersects. Converse is true.
20 If $\frac{1}{a} \not> \frac{1}{3}$, then $a \le 0$ or $a \ge 3$. True.
21 a use SSS test
b matching angles of congruent triangles equal, alternate angles
c matching angles of congruent triangles equal, alternate angles
d two pairs of opposite sides parallel
22 Proof

3.01

1 D
2 B
3 456 976
4 67 600
5 $26^5 \times 10^4$
6 260
7 $26^{10} \times 10^{15}$
8 1000
9 1 000 000
10 300
11 64
12 Yes
13 Yes
14 6
15 6840
16 360
17 7 880 400
18 210
19 271 252 800
20 $\frac{3}{10\,000}$
21 a 84 b $\frac{1}{84}$
22 a 10 000 000 b 1000
23 $\frac{1}{67\,600\,000}$
24 $\frac{1}{5184}$
25 a 9900 b $\frac{1}{9900}$
26 $\frac{1}{720}$
27 7
28 a 60 b 720 c 30 240
29 64
30 17 576 000. More number plates are possible with 3 letters.
31 72
32 36
33 a 25 b 20

3.02

1 36
2 7
3 9
4 22
5 960
6 a 84 b 504
7 a 840 b 480 c 480
d 480 e 240 f 720

8 a 1680 b 4096 c 720 d 720
e 120 f 1560 g 18 000
9 a 120 b 48
10 a 18 b 6 c 4
d 10 e 8 f 10
11 a 1680 b 6720 c 5400
d 840 e 4200
12 42
13 a 24 b 256
14 a 5 b 6 c 12
15 288

3.03

1 a 5 b 14
2 3
3 366 unless it is a leap year, when it is 367.
4 27
5 16
6 17
7 21
8 169
9 16
10 The 200 numbers can be divided into 100 pairs such that the second is double the first: 1\2, 2\4, 3\6, 4\8, 5\10, …, 100\200. By the principle, from 101 numbers between 1 and 200 there must be at least 2 that belong to the same pair, so there is at least one number that is double one of the others.
11 The 200 numbers can be divided into two separate groups: 66 pairs such that one is triple the other: 1\3, 2\6, 3\9, …, 66\198 and another 68 that are not triple any other number from 1 to 200. If 66 + 1 = 67 of the first group are placed into 66 boxes, then at least one box must contain a pair. The other 68 can be placed separately, making 67 + 68 = 135 numbers that must be chosen to ensure that at least one number is triple another.
12 If all the integers are the same value, then that value is the average and it is trivially true. Otherwise, assign the integers to two groups: those with values equal to the greatest value, and all others. By the pigeonhole principle, there must be an integer in each group, otherwise, revert to all values being the same. The average of the values lies between the greatest value and the least value, so it must be less than the greatest value. Hence, combining both parts, one of the integers must be greater than or equal to the average value.
13 Consider, say, Mary. There are 5 others, so by the principle she is either friends with 3 of them or there are 3 that are strangers to her. Of 3 friends, either 2 are friends with each other and Mary or the 3 are mutual strangers. Of 3 strangers, either 2 are strangers to each other and Mary, or the 3 are mutual friends. Either way, there are either 3 mutual strangers or 3 mutual friends.
14 Suppose there are n people. If they all have at least one friend, then they all have from 1 to $(n - 1)$ friends, so by the principle at least 2 must have the same number of friends. If someone has no friends, then there are $(n - 1)$ left. If one of them has no friends, then there are 2 people in the whole group with no friends. Otherwise, the remaining $(n - 1)$ people have between 1 and $(n - 2)$ friends, so by the principle there must be 2 of the $(n - 1)$ with the same number of friends.

3.04

1 B
2 C
3 C
4 E
5 a 120 b 362 880 c 3 628 800
d 24 e 479 001 600
6 5040
7 4.03×10^{26}
8 a 6 b 60 c 42
d 362 880 e 1680 f 120
g 110 h 30 240 i 592 620
j 240 240
9 a $n = 10$ b $n = 3$ c $n = 15$
d $n = 29$ e $n = 5$ f $n = 10$
10 24
11 5040
12 720
13 a 18 b 192
14 a 720 b 6840 c 24 360
15 a 24 b 720 c $\frac{1}{30}$
16 $\frac{1}{8}$
17 $(n+1)! - n! = (n+1) \times n! - n! = n!(n+1-1)$
$= n \times (n-1)! \times n = (n-1)!\, n^2$

3.05

1 B
2 a 300 b 1080
3 a 96 b 261 c 93
4 a 720 b 4320 c 720
5 2880
6 a 3 628 800 b 362 880 c 28 800
7 a 6 b 720 c 5040
d 362 880 e 3 628 800
8 362 880
9 a 720 b 240 c 480 d 144
10 $\frac{2}{9}$
11 86 400
12 a 181 440 b 19 958 400
c 20 160 d 1 814 400
e 239 500 800
13 205 920
14 a 120 b 150

15 a 20! b 5! 8! 7! 3!
16 a 40 320 b 30 240 c 21 600
17 4680

3.06

1 a 60 480 b 2520 c 907 200
d 151 200 e 60 f 453 600
g 1 995 840 h 3360 i 83 160
j 145 297 152 000

2 a 60 b 48 c 36 d $\frac{1}{5}$
3 48
4 100
5 151 200
6 5040
7 About 4.22×10^{15}
8 12 600
9 a 6840 b 8000 c 7980
10 About 8.72×10^{10}, as you could start anywhere around the walls when viewing and in an exhibition, none of the paintings would be the same.
11 a 64 b 15 c $\frac{15}{64}$
12 a $\frac{3}{8}$ b $\frac{7}{64}$ c $\frac{5}{16}$
d $\frac{7}{64}$ e $\frac{184\,756}{1\,048\,576} \approx 0.176$

3.07

1 $\frac{1}{336}$
2 $\frac{1}{32\,760}$
3 a $\frac{1}{120}$ b $\frac{1}{210}$ c $\frac{1}{150}$ d $\frac{1}{7350}$
4 $\frac{1}{30}$
5 $\frac{1}{15\,400}$
6 $\frac{1}{140}$
7 $\frac{72}{325}$
8 $\frac{1}{56}$
9 $\frac{1}{3\,628\,800} \approx 0.000\,000\,28$

CHAPTER 3 REVIEW

1 E
2 E
3 B
4 D
5 D
6 456 976 000
7 20 160
8 $\frac{1}{128}$
9 32
10 14 400
11 6
12 9
13 9
14 362 880
15 30 240
16 720
17 256
18 576
19 420
20 $\frac{1}{4080}$
21 24
22 20 160
23 Separate the speakers by brand. Then if $12 \times 8 - 8 + 1 = 89$ speakers are placed in each group, by the strong pigeonhole principle (with $m_i = 12$) at least one group must contain at least 12 speakers.
24 64
25 1260

MIXED REVISION 1

Multiple choice

1 D
2 E
3 C
4 B
5 C
6 C
7 B
8 B
9 D

Short answer

1 (8.54, 290.6°)
2 Sample answers:
a Monkeys, people, dolphins or whales
b Any number between 0 and 1, e.g. $0.5^2 = 0.25$.
c A kite with angles of 90°, 110°, 50° and 110° is not a rectangle.
3 a 18 b $\frac{2}{19}$
4 (2, 4) or 2**i** + 4**j**
5 Sample answers:
Inductive logic uses examples to reach a conclusion:
e.g. Holdens have four wheels
Falcons have four wheels
Toyotas have four wheels
Thus all cars have four wheels
Deductive logic uses logic to proceed from one (true) statement to another:
e.g. Reptiles lay eggs with soft shells
Goannas are reptiles
Thus goannas lay eggs with soft shells
6 2184

Application

1 (77.9, 81.5°), 77.9 km at the bearing 081.5°
2 About 39.3 knots at a bearing of 167.4°.
3 Use a counter example, of which 10 is the first: $10^2 + 10 + 11 = 121 = 11 \times 11$.
4 Proof
5 $\frac{1}{245\,157} \approx 0.000\,004$
6 $\frac{6}{77}$

4.01

1	a	12.12	b	5.5	c	28.28	d	−8.87
	e	0	f	30	g	−35.88	h	63.74
2	a	3	b	24.25	c	28.28	d	−45.55
	e	0	f	27	g	23.79	h	108.12
3	a	−240.99	b	−102.31	c	32.47	d	80.8
	e	4.89	f	59.06	g	−83.54	h	−61.52
4	a	−258.85	b	−141.38	c	233.41	d	−26.31
	e	−15.35	f	168.77	g	−39.6	h	−28.17
5	a	16	b	56	c	8		
	d	18	e	−29	f	−60		

4.02

1	a	10	b	12	c	5
	d	2	e	18		
2	a	44.50	b	46.36	c	57
3	a	19	b	23	c	28
	d	9	e	18	f	5
4	a	5	b	14	c	4
	d	20	e	2	f	−11
5	a	−14	b	0	c	−96
	d	−30	e	−85	f	73
6	a	27.59	b	11.31	c	11.19
	d	−8.12	e	31.89	f	−8.03
7	a	14.25°	b	8.13°	c	167.9°
	d	135°	e	147.53°	f	145.49°
8	a	49.4°	b	97.7°	c	170.84°
	d	99.73°	e	147.53°	f	31.33°

9 $\cos(\theta) = 0$, so $\theta = 90°$.
10 $\cos(A - B) = [\cos(A), \sin(A)] \cdot [\cos(B), \sin(B)]$
$= \cos(A)\cos(B) + \sin(A)\sin(B)$

4.03

1–4 Demonstration by substitution.
5–9 Proofs

4.04

1 B
2–3 For each pair of vectors **a** and **b**, **a** · **b** = 0.
4 For **a**, **c**, **f**, **a** · **b** = |**a**||**b**|, for **b**, **d**, **e**, **a** · **b** = −(|**a**||**b**|).
5 For **a**, **c**, **e**, **f**, **a** · **b** = |**a**||**b**|, for **b**, **d**, **a** · **b** = −(|**a**||**b**|).
6 a Neither
b Parallel (in opposite directions)
c Neither
d Parallel
e Perpendicular
f Neither
7 $a = 2\frac{1}{4}$
8 $b = 8$
9 Maximum value = 20 when **m** and **n** are parallel. Minimum value = −20 when **m** and **n** are parallel but in opposite directions.
10 49.4°

4.05

1 D
2 B
3 C
4 B
5 16.18
6 12.61
7 −2.21
8 29.67
9 0 (perpendicular)
10 19 (parallel)
11 $4\frac{3}{5}$
12 $-5\frac{9}{13}$
13 $\frac{31}{\sqrt{53}}$
14 $-\frac{9}{\sqrt{10}}$
15 $\frac{15}{\sqrt{5}}$

4.06

1 A
2 D

3	a	i	3.04**i** + 2.28**j**	ii	0.96**i** − 1.28**j**
	b	i	0.224**i** + 0.168**j**	ii	0.276**i** − 0.368**j**
	c	i	16**i** + 12**j**	ii	0
	d	i	−0.4**i** − 0.3**j**	ii	0
	e	i	−4.64**i** − 3.48**j**	ii	−0.36**i** + 0.48**j**
	f	i	−8**i** − 6**j**	ii	0
4	a	i	−6.92**i** − 4.62**j**	ii	6.92**i** − 10.38**j**
	b	i	2.88**i** − 0.58**j**	ii	−2.88**i** − 14.42**j**
	c	i	−15**j**	ii	0
	d	i	0	ii	−15**j**

5 About 1008 N at 13.4° towards the 500 N side of the 400 N force.
6 About 977 N each.
7 The component of the wind in the direction of the sprint is about 3.16 km/h.
8 About 1097 N parallel and 8933 N perpendicular to the slope.
9 About 1.25 N parallel and 5.87 N perpendicular, so the friction was about 0.55 N.
10 About 96 N
11 About 743 N

9780170250276

4.07

1 C
2 B
3 a 500 J b 400 J c 600 J
 d 1200 J e 50 000 J f 900 J
4 a About 22.4 N b About 40.3 J
5 a 16 J b −10 J c 75 J
 d −29 J e 0 f 0
6 7071 N
7 About 9888 J
8 72 J
9 a 76.53 N at 9° to the 50 N force.
 b i 458.79 J ii 231.8 J iii 257.1 J
 c 488.9 = 231.8 + 257.1, but because of the angles, the contributions of the 30 N and 50 N forces are almost the same.
10 a 8567 N perpendicular and 749.5 parallel to the slope, so about 250 N each.
 b 89.94 kJ
11 7713 J
12 2.88 MJ
13 28.16 kJ
14 Proof
15 Proof

4.08

1 C
2 A
3 About 9.6 km at 089.8°.
4 520 m
5 About 181.8 knots at 066.5°
6 About 7.8 knots at 120.6°.
7 About 9.3 knots at 216.1°.
8 About 124.9° and 10.1 knots.
9 About 314.9° and 119.8 knots.
10 About 12:31:39 p.m.
11 About 9.8 knots.
12 9.434 knots at 32.01° upstream from the heading.
13 103.2 knots at 215.6°.
14 028.17°, to give a ground speed of 105 knots and an ETA of 1:35 p.m.

CHAPTER 4 REVIEW

1 A
2 C
3 B
4 C
5 B
6 A
7 E
8 D
9 B
10 a 20.54 b 14.65
11 a −32 b −26
12 63.4°
13 37.5
14 38
15 a −55 b −41 c 6
16 a 19.86 b −48.75
17 a 10.3° b 176.8°
18 Demonstration by substitution.
19 a, b Scalar product = 0
20 a $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|$ b $\mathbf{a} \cdot \mathbf{b} = -|\mathbf{a}||\mathbf{b}|$
21 a Perpendicular b Neither
 c Parallel but opposite in direction
22 17.10
23 7.23
24 a $1.10\mathbf{i} + 2.76\mathbf{j}$ b $-3.10\mathbf{i} + 1.24\mathbf{j}$
25 181.26 N (north) and 84.52 N (east)
26 a 300 J b 1904 J
27 Proof
28 123.69°
29 28.81 N at 10.7° to the right.
30 1476 N (parallel) and 8371 N (perpendicular).
31 675.3 N, 663.58 N
32 3236 J
33 About 149.5° and 7.7 knots
34 225° and 95 knots
35 Proof

5.01

1 a 15 b 1 c 25
 d $\frac{n(n-1)(n-2)}{6}$ e $\frac{(n+1)n}{2}$
2 a 56 b 210 c 495
 d 1287 e 1287
3 a 28 b 84 c 462
 d 5005 e 38 760
4 a $n = 4$ b $n = 2$ c $n = 10$
 d $n = 6$ e $n = 12$ f $n = 16$
5 2 598 960
6 a 45 b $\frac{n(n-1)}{2}$
7 10
8 252
9 15 504
10 210
11 296 010
12 792
13 3 838 380
14–15 Proofs

5.02

1 a 15 504 b 816
2 60
3 7260
4 a 120 b 112
5 935
6 a 45 b 120
7 a 100 947 b 462 c 924 d 36 300
 e 26 334 f 74 613 g 27 225
8 $105

9 a 2 042 975 b 55 c 462 462 d 30 030

10 a 3003

b i 2450 ii 588 iii 56 iv 1176

11 a 1.58×10^{10} b 286

c 15 682 524 d 5 311 735

e 12 271 512

12 a 395 747 352 b 32 332 300

c 4 084 080 d 145 495 350

e 671 571 264

13 a 170 544 b 36 c 20 160

d 17 640 e 6300

14 a 7 b 27 132 c 13 860

d 20 790 e 27 720

15 a 5 b 360 c 126

16 a 715 b 495 c 330 d 1287

5.03

1–5 Proofs

6 $(x + y + z)^3 = x^3 + y^3 + z^3 + 3x^2y + 3x^2z + 3xy^2 + 3xz^2 + 3y^2z + 3yz^2 + 6xyz$

7–8 Proofs

5.04

1 67

2 627

3 700

4 713

5 600

6 600

7 642

8 6600

9 188

10 440

11 978

12 43

5.05

1 a 44 b $\frac{4}{11} \approx 36\%$

2 $\frac{2}{3}$

3 $\frac{1}{26} \approx 0.03846$

4 a $\frac{1}{6}$ b $\frac{1}{3}$

5 a $\frac{1}{24} \approx 0.041\ 67$ b $\frac{3}{8} = 0.375$

6 a $\frac{1}{24} \approx 0.041\ 67$ b $\frac{1}{4} = 0.25$

c $\frac{3}{8} = 0.375$ d $\frac{1}{2} = 0.5$

5.06

1 500

2 a 22 b 84 c 270 d 690

3 1440

4 1422

5 4020

6 a $2! \times 19! \approx 2.433 \times 10^{17}$

b $3! \times 18! \approx 3.842 \times 10^{16}$

c $4! \times 17! \approx 8.537 \times 10^{15}$

d $20! - (6 \times \mathbf{a} - 4 \times \mathbf{b} + \mathbf{c})! \approx 1.118 \times 10^{18}$

7 ${}^{12}C_4 \times 7! \times 3! = 14\ 968\ 800$

5.07

1 $\frac{32}{663} \approx 0.048\ 27$

2 a $\frac{7}{102} \approx 0.068\ 63$ b $\frac{7}{306} \approx 0.022\ 88$

c $\frac{7}{17} \approx 0.4118$

3 $\frac{6}{4165}$

4 a $\frac{33}{16660}$ b $\frac{1}{899}$ c $\frac{7371}{3\ 235\ 501} \approx 0.022\ 278$

5 $\frac{1}{3}$

6 a 0.0976 b About 0.9785

c 28

7 a $\frac{1}{3}$ b $\frac{17}{24}$ c $\frac{9}{24}$

8 0.7

9 1

10 $\frac{1}{1200} \approx 0.000\ 83$

11 a $\frac{1}{253} \approx 0.003\ 95$ b $\frac{7}{253} \approx 0.0277$

12 a $\frac{1}{35} \approx 0.028\ 57$ b $\frac{1}{21} \approx 0.047\ 62$

c $\frac{10}{231} \approx 0.043\ 29$ d $\frac{1}{13} \approx 0.076\ 92$

CHAPTER 5 REVIEW

1 C

2 B

3 D

4 B

5 E

6 78

7 420

8 a 2600 b 4032

9 390

10 $\frac{23}{110} \approx 0.209$

11 150

12 440

13 $\frac{80}{221}$

9780170250276

14 $\frac{n}{r}\left({}^{n-1}C_{r-1}\right)$

$=\frac{n}{r}\times\frac{(n-1)!}{[(n-1)-(r-1)]!(r-1)!}$

$=\frac{n(n-1)!}{(n-1-r+1)\times!r(r-1)!}$

$=\frac{n!}{(n-r)!r!}$

$={}^{n}C_{r}$ **QED**

15 ${}^{n}C_{r}=\frac{n(n-1)\cdots(n-r+1)}{r!}$.

For n prime and $0 < r < n$, there is a factor of n in the numerator, and no factor in the denominator divides n, as it is prime. Thus, there is a factor of n in the number ${}^{n}C_{r}$. **QED**

16 a 2898 b 21 252 c 33 649

17 568

6.01

1 a $x = 68°$ b $y = 97°$
c $\beta = 40°$ d $x = y = 39°$

2 a $x = 112°, y = 56°, z = 34°$ b $x = 49°$
c $x = 166°, y = 7°$ d $x = 62°, \beta = 31°$

3 a $x = 5°$ b $y = 102°$

4 $x = 30°$ (∠ at centre is double the ∠ at the circumference)
$y = 75°$ (∠ sum of isosceles △)

5 $360° – x = 2 \times 110°$ (∠ at centre is double the ∠ at the circumference)
$x = 140°$
$y = 70°$ (similarly)

6 $\angle OAC = 30°$ (Base ∠s of isosceles △)
$\angle BAO = 25°$ (similarly)
$\therefore \angle CAB = 30° + 25°$
$= 55°$
$x = 2 \times \angle CAB$ (∠ at centre is double the ∠ at the circumference)
$= 2 \times 55°$
$= 110°$

7 Obtuse $\angle BOD = 2\theta$ (∠ at centre double ∠ at circumference)
Reflex $\angle BOD = 360 – 2\theta$ (angle of revolution)
$\angle BCD = \frac{1}{2}\angle BOD$ (∠ at centre double ∠ at circumference)
$= \frac{1}{2}(360 – 2\theta)$
$= 180 – \theta$
So $\angle BCD$ and $\angle DAB$ are supplementary (add to 180°)

6.02

1 a $x = 18°$ b $\alpha = 83°, \beta = 42°$
c $x = 97°$ d $m = 136°$

2 a $a = 65°$ (∠ at centre = twice ∠ at circumference standing on arc ED)
$b = 65°$ (∠s at circumference standing on arc ED are equal)
b $\angle PRQ = 41°$ (∠s at circumference standing on arc PQ are equal)
$w = 33°$ (∠sum of $\triangle QRT$)
c $\angle NKM = 28°$ (∠s at circumference standing on arc NM are equal)
$a = 56°$ (exterior ∠ of $\triangle NPK$ = sum of 2 interior opposite ∠s)
d $\angle DEC = 141°$(∠sum of $\triangle DEC$)
$m = 141°$ (vertically opposite ∠s)
$\angle ACB = 31°$ (∠s at circumference standing on arc AB are equal)
$x = 110°$ (exterior ∠ of $\triangle BEC$ = sum of 2 interior opposite ∠s)

3 a $\angle DCE = \angle ACB$ (vertically opposite ∠s)
$\angle EDC = \angle BAC$ (∠s in the same segment)
$\angle DEC = \angle ABC$ (similarly)
∴ Since all pairs of ∠s are equal,
$\triangle ABC \,|||\, \triangle DEC$.
b $x = 5.5$ cm

4 $\angle STV = \angle WUV$ (∠s in the same segment)
$\angle TSV = \angle UVW$ (similarly)
$\angle TVS = \angle UVW$ (vertically opposite ∠s)
∴ Since all pairs of angles are equal,
$\triangle STV \,|||\, \triangle WUV$.
$x = 2.4$ cm

5 $\angle ECB = 33°$ (∠s in the same segment)
$\angle EBC = 180° – 114° + 33°$ (∠sum of $\triangle EBC$)
$= 33°$
$\therefore \angle ECB = \angle ADE$
These are equal alternate angles.
$\therefore AD \,||\, BC$

6 Proof

6.03

1 a $\theta = 29°$
b $x = 10$ cm
c $x = 15°, y = 150°, z = 75°$
d $x = 47°, y = 43°, z = 94°$

2 a $x = 55°, y = 43°$
b $x = y = 32°, z = 58°, v = 32°, w = 17°$
c $x = 57° 30', y = 32° 30'$
d $x = 75°, y = 77°, z = 13°$

3 $\angle ABC = 90°$ (∠ in semicircle)
$\therefore \angle BAC = 90°$ (∠sum of $\triangle ABC$)
$= 61°$
$\therefore x = 61°$ (∠s in the same segment)

4 Proof.
Prove $RS = SP$. ∴ S is centre of circle.

5 Proof. Use isosceles $\triangle OAC$ and $\triangle OBC$.

6 $\angle B = 90°$ (∠in semicircle)
$AC^2 = AB^2 + BC^2$
$= 6^2 + 3^2$
$= 36 + 9$
$= 45$

$\angle AC = \sqrt{45}$
$= 3\sqrt{5}$
Radius $= \frac{1}{2}AC$
$= \frac{3\sqrt{5}}{2}$ cm

7 a $x = 52°, y = 76°$
b $AC = BD$ (equal diameters)
Diagonals are equal so $ABCD$ is a rectangle.
$\therefore AD = BC$ (opposite sides of a rectangle)

8 a $\angle AOB = 90°$ (given)
$\angle ABC = 90°$ (angle in semi-circle)
$\therefore \angle AOB = \angle ABC$
$\therefore \angle A$ is common
$\therefore \triangle AOB \,|||\, \triangle ABC$ (AAA)
(Note 2 pairs of angles equal means 3 pairs will be equal by angle sum of triangle.)
b $AO = BO$ (equal radii)
$AB = \sqrt{r^2 + r^2}$
$= \sqrt{2r^2}$
$= \sqrt{2} \times \sqrt{r^2}$
$= \sqrt{2}r$
By similar triangles
$\frac{AO}{AB} = \frac{BO}{BC}$
But $AO = BO$ so $AB = BC$
So $BC = \sqrt{2}\, r$

6.04

1 a $\theta = 32°$ b $x = 8$ cm c $\theta = \alpha = 68°\ 30'$
d $\theta = 31°$ e $x = 9$ mm f $\theta = 45°$

2 a $x = 5$ cm b $y = 15$ cm c $x = 42°$
d $z = 90°$ e $x = 6$ m, $y = 3$ m
f $m \approx 13.4$ cm

3 $\sqrt{41}$ cm

4 144 mm

5 25.6 cm

6 $CE = \sqrt{11.5^2 - 6.9^2}$
$= 9.2$
$CD = 2 \times 9.2$ (perpendicular from O bisects chord)
$= 18.4$
$= AB$

7 $OB = 8.3$ cm

8 Proof

9 Proof

6.05

1 a $x = 2.4$ m b $x \approx 10.3$ m
c $y \approx 5$ cm d $a = 14$

2 a $x = 4.7$ m, $y = 1.8$ m
b $x \approx 4.4$ m, $\alpha = 78°, \beta = 38°, \theta = 64°$

3 a $x = 2$ b $y = 4$ c $z = 6$

4 a $\angle ECD = \angle ACB$ (vertically opposite angle)
$\angle A = \angle E$ (angle in same segment)
$\therefore \triangle ABC \,|||\, \triangle CDE$ (AAA)
b By similar triangles
$\frac{AC}{CE} = \frac{BC}{CD}$
$AC \times CD = BC \times CE$

6.06

1 a $\theta = 47°$ b $x = 5$ m
c $y = 11.3$ cm d $x = y = 26°$
e $a = 64°, b = 32°$ f $\theta = 57°$
g $p = \sqrt{145} \approx 12$ cm h $y = 10$ mm
i $x \approx 5.79$ cm j $x = 33°, y = 33°$

2 a $x = 67°$ b $y \approx 7.5$ cm
c $x = 63°, y = 126°$ d $x = 8.9$ m, $y \approx 6.2$ m
e $x = 98°$ f $x = 57°, y = 57°$
g $x = 72°, y = 15°$ h $x = 61°, y = 70°, z = 52°$

3 a $x = 26°, y = 74°, z = 48°$
b $x = 68°, y = 44°, z = 68°$
c $x = y = z = 45°$
d $x = 70°, y = 31°$
e $x = 20°, y = 57°, z = 103°$
f $x \approx 5.4$ cm
g $x \approx 7.1$ cm
h $x = 77°, y = 13°$
i $x \approx 1.2$ cm, $y \approx 2.1$ cm
j $x = 55°, z = 57°$

4 $AB \approx 13$ m

5-6 proofs

6.07

1 a $x = 107°, y = 94°$
b $\theta = 134°, \gamma = 90°$
c $x = 112°, y = 112°, z = 68°$
d $x = 92°, y = 114°$
e $\beta = 73°, \alpha = 107°, \gamma = 107°$
f $x = 141°, y = 63°$
g $x = 65°, y = 43°$
h $w = 89°, x = 86°, y = 54°, z = 35°$
i $w = 69°, x = 111°, y = 82°, z = 98°$
j $x = 118°$

2 a $x = 62°, y = 31°$ b $x = 75°, y = 105°$
c $x = 88°, y = 65°$ d $x = 62°, y = 82°, z = 36°$
e $x = 90°, y = 113°$ f $x = 38°, y = 71°$
g $x = 85°, y = 95°$ h $x = 48°, y = 78°$
i $x = 107°, y = 73°$
j $a = 81°, b = 55°, c = 83°, d = 16°, e = 28°$

3 a $\angle A = 180° - 58°$ ($\angle A$ and $\angle B$ co-interior angles, $AD \,||\, BC$)
$\angle D = 180° - 58°$ ($\angle C$ and $\angle D$ co-interior angles, $AD \,||\, BC$)
So $\angle A = 180° - \angle C$ and $\angle D = 180° - \angle B$.
Since opposite angles are supplementary, $ABCD$ is a cyclic quadrilateral.
b $\angle B = \angle D = 90°$ (given)
$\therefore \angle B = 180° - \angle D$.
Let $\angle A = x$

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$\angle C = 360° - 90° + 90° + x$ ($\angle$sum of quadrilateral)
$= 360° - 180° - x$
$= 180° - x$
$= 180° - \angle A$
Since opposite angles are supplementary, *ABCD* is a cyclic quadrilateral.

c $\angle CDA = 180° - \theta$ (straight angle)
$\therefore \angle B = 180° - \angle CDA$
Let $\angle A = x$
$\angle C = 360° - 90° + 90° + x$ ($\angle$sum of quadrilateral)
$= 360° - 180° - x$
$= 180° - x$
$= 180° - \angle A$
Since opposite angles are supplementary, *ABCD* is a cyclic quadrilateral.

d $\angle DAB = 90°$ (tangent $\perp$ radius)
$\angle DCB = 90°$ (tangent $\perp$ radius)
Now $\angle DAB + \angle DCB = 90° + 90°$
$= 180°$
Since opposite angles are supplementary, *ABCD* is a cyclic quadrilateral.

4 Proof

6.08

1 a $x = 58°, y = 116°, z = 58°$ b $x = 5$ mm

2 a $x = 10$ cm b $x = 64°, y = 26°$
c $x = 13$ cm d $x = 27°, y = 54°$
e $y = 5$ cm f $x = 32°, y = 7°$
g $x = 72°, y = 42°$ h $x = 35°, y = 90°$
i $m = 23°, n = 67°, p = 67°, q = 23°$
j $x = 71°, y = 62°$

3 $\therefore \angle OAB = 90°$ (tangent $\perp$radius)
$z = 90° - 48°$ ($\angle$sum of $\triangle ABC$)
$= 42°$
$OA = OC$ (equal radii)
$\therefore \angle OAC = \angle OCA = y$ (base $\angle$s of isosceles $\triangle$)
$y = (180° - 48°) \div 2$ ($\angle$ sum of $\triangle OAC$)
$= 66°$
$\angle ACD = 180° - \angle AED$ (opposite $\angle$s of cyclic quad.)
$\therefore y + u = 180° - 62°$
$66° + u = 118°$
$u = 52°$
$\angle BAC = \angle OAB - \angle OAC$
$\therefore x = 90° - 66°$
$= 24°$
$v = \frac{1}{2}\angle AOC$ ($\angle$ at centre twice $\angle$ at circumference)
$= \frac{1}{2} \times 48°$
$= 24°$

4 21 cm

5 $AC^2 + BC^2 = 3.9^2 + 5.2^2$
$= 42.25$
$AB^2 = 6.5^2$
$= 42.25$
$\therefore AB^2 = AC^2 + BC^2$
$\therefore \angle ACB = 90°$ (by Pythagoras' theorem)
$\therefore A$ lies on a diameter of the circle. (tangent $\perp$radius)

6 a $x = 72°, y = 121°$ b $x = 63°, y = 63°$

7 $\angle DAC = 63°$ ($\angle$sum of $\triangle DAE = 180°$)
$\therefore \angle DAC = \angle DBC = 63°$
$\therefore A, B, C, D$ are concyclic (equal $\angle$s at circumference standing on chord *CD*)

8 $OA = r$
$AC = \frac{x}{2}$ (perpendicular from the centre bisects a chord)
$OC = \sqrt{r - \left(\frac{x}{2}\right)^2}$ (Pythagoras' theorem)
$= \sqrt{r^2 - \frac{x^2}{4}}$
$= \sqrt{\frac{4r^2}{4} - \frac{x^2}{4}}$
$= \sqrt{\frac{4r^2 - x^2}{4}}$
$= \frac{\sqrt{4r^2 - x^2}}{2}$
$CD = r + \frac{\sqrt{4r^2 - x^2}}{2}$
$= \frac{2r + \sqrt{4r^2 - x2}}{2}$

9 $\angle BDE = \angle ABD + \angle BAD$
$\therefore 2\alpha = \angle ABD + \alpha$
$\alpha = \angle ABD$
$\therefore \triangle BAD$ is isosceles with $AD = BD$.
$\angle CDE = \angle ACD + \angle CAD$ (ext. $\angle$ of $\triangle BAD$)
$\therefore 2\beta = \angle ACD + \beta$
$\beta = \angle ACD$
$\therefore \angle CAD$ is isosceles with $AD = CD$.
$\therefore AD = BD = CD$
So a circle can be drawn through *A*, *B* and *C* with centre *D*.

10 Let $\angle ODC = x$ and $\angle OAB = y$.
Then you can find all these angles (giving reasons).
$\angle AOC + \angle COB + \angle BOD + \angle AOD = 360°$
($\angle$of revolution)
$90° - y + x + \angle COB + y + 90° - x + \angle AOD = 360°$
$\angle COB + \angle AOD + 180° = 360°$
$\therefore \angle COB + \angle AOD = 180°$

11 Let *ABCD* be a quadrilateral with opposite angles supplementary.
i.e. $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$.
Assume the points are not concyclic. Draw a circle through *A*, *B* and *C*, cutting *CD* at *E*.
Now *ABCE* is a cyclic quadrilateral, so $\angle AEC + \angle B = 180°$. (opposite $\angle$s are supplementary)
Also, $\angle D + \angle B = 180°$ (given)
$\angle D = \angle AEC$
These are equal corresponding angles, so $DA \parallel EA$ (this is impossible!)
$\therefore A, B, C$ and *D* must be concyclic.
$\therefore ABCD$ is a cyclic quadrilateral.

CHAPTER 6 REVIEW

1 C
2 E
3 B
4 D
5 E
6 C
7 D
8 $\theta = 56°$
9 $y = 2.3$ mm
10 $x = 7.2$ m
11 $x = y = 12$ cm
12 $z = 19°$ (∠s in same segment)
$y = 180° - (131° + 19°)$ (∠sum of Δ)
$= 30°$
$x = 30°$ (∠s in same segment)
13 $x = 10$ cm
14 $\alpha = 3°, \beta = 44°, \gamma = 136°$
15 $a = \frac{1}{2} \times 100°$ (∠ at centre twice ∠ at circumference)
$= 50°$
$\angle OCA = 90°$ (tangent perpendicular to radius)
$\therefore b = 90° - 83°$
$= 7°$
$OC = OE$ (equal radii)
$\therefore \triangle OCE$ is isosceles.
$\therefore \angle OCE = \angle OEC = c$
$2c + 100° = 180°$ (∠ sum of Δ)
$c = 40°$
Reflex $\angle COE = 360° - 100°$ (∠ of revolution)
$= 260°$
$d = 360° - (260° + 50° + 7°)$ (∠ sum of quadrilateral)
$= 43°$
16 17 cm
17 5.3 m
18 $a = 101°, b = 98°$
19 $\alpha = 61°, \beta = 29°$
20 14.9 cm
21 $x = 4.9$ m
22 18 cm
23 $\alpha = 127°, \beta = 53°$
24 $\angle D = 180° - (80° + 53°)$ (∠sum of Δ)
$= 47°$
$\therefore y = 47°$ (∠s in same segment)
$x = 47°$ (∠s in alternate segment)
25 $x = 55°, y = 56°, z = 54°$
26 $\angle C$ is common.
$\angle A = \angle CBD$ (∠s in alternate segment)
$\therefore \triangle BCD \,|||\, \triangle ABC$ (AAA)
27 a $\angle OCB = \angle OCA = 90°$ (given)
$OA = OB$ (equal radii)
OC is common.
$\therefore \triangle OAC \equiv \triangle OBC$ (RHS)
b $AC = BC$ (corresponding sides in ≡ △s)
$\therefore OC$ bisects AB.
28 Proof
29 Proof
30 Proof

MIXED REVISION 2

Multiple choice

1 D
2 A
3 B
4 A
5 A
6 E
7 E
8 E
9 B

Short answer

1 a 14 b 27.58
2 20 have all three.
3 $\angle LMN = 90°$ (angle in a semicircle)
$x = 41$ (complementary angles in $\triangle LMN$)
4 3.13
5 $\frac{496}{14\,259} \approx 3.479\%$
6 $IK \times KG = HK \times KF$ (product of intervals on chord IG = product of intervals on chord HF)
$y \times 12 = 18 \times 8$
$12y = 144$
$y = 12$

Application

1 About 74.1°
2 1.839 MJ
3 $\frac{264}{1885}$
4 1287
5 $3p = 360$ (revolution)
$p = 120$
$\therefore \angle BAD + \angle BCD = p + 60$
$= 120 + 60$
$= 180$
$\therefore ABCD$ is cyclic (opposite angles of quadrilateral supplementary).
6 Perimeter $= 2x + 2y + 40 = 2(16) + 40 = 72$ cm

7.01

1 D
2 a $2a + 1 + 2b + 1 = 2(a + b + 1)$
b $2a(2b + 1)$ or $2(ab + a)$
3 E
4 12, 18
5 a $1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 + 1024 + 2048 + 4096 + 8191 + 16\,382 + 32\,764 + 65\,528 + 131\,056 + 262\,112 + 524\,224 + 1\,048\,448 + 2\,096\,896 + 4\,193\,792 + 8\,387\,584 + 16\,775\,168 = 33\,550\,336$
b $2^{12}(2^{13} - 1)$

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6 $1 + 2 + 4 + 8 + 16 + 32 + 37 + 74 + 148 + 296 + 592 = 1210$, $1 + 2 + 5 + 10 + 11 + 22 + 55 + 110 + 121 + 242 + 605 = 1184$
7 No, as 2 is prime.
8 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.
9 a No b Yes
10 Yes
11 6
12 a 2^{16} b $2^{23} - 1$ is not prime

INVESTIGATION: PLAYING WITH INTEGERS

1 a i 65 ii 17 iii 31
b i 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97
ii 7919 iii 107 iv No
2 a i 5 ii 4 iii Yes
b i show that ii $2^4 \times 31$, $2^6 \times 127$
c $2^{p-1} \times$ prime, use $S_n = \frac{a(r^n - 1)}{r - 1}$, 2^{p-1} will have p factors, $1, 2, 2^2, \ldots, 2^{p-1}$, $S_p = \frac{1(2^p - 1)}{2 - 1} = 2^p - 1$, there will be p remaining terms which are multiples of the prime number, $b, 2b, \ldots, 2^{p-1} \times b$, $S_p^* = (2^p - 1)b$, $S_p + S_p^* = (2^p - 1) + (2^p - 1)b = 2 \times 2^{p-1} \times b = 2^p b$, $2^p - 1 - b = 0$, $b = 2^p - 1$
d

p	Is $2^p - 1$ prime?	Is $2^{p-1}(2^p - 1)$ a Perfect number?
2	Yes	Yes
3	Yes	Yes
5	Yes	Yes
7	Yes	Yes
11	No	No
13	Yes	Yes
17	Yes	Yes
19	Yes	Yes
23	No	No
29	No	No
31	Yes	Yes

e i No, the only proper divisor of a prime is 1, so there is no prime equal to the sum of its proper divisors.
ii If a Mersenne prime exists, a corresponding perfect number exists.
f 47 (the same as the number of Mersenne primes)

7.02

1 a $\frac{1}{3}$
b $(2a + 1)(2b + 1) = 4ab + 2a + 2b + 1$
$= 2(2ab + a + b) + 1$
2 a i Yes ii No iii No iv No
b i Closed ii $-1 - (-2) = 1$
iii $-1 \times (-2) = 2$
iv $\frac{-1}{-2} = 0.5$
3 No, $1 \div 2$
4 a No b Yes
5 a 2 b 0
6 $2k + 1 + 2k + 3 = 4k + 4 = 4(k + 1)$
7 $(k + 1)^2 - k^2 = k^2 + 2k + 1 - k^2 = 2k + 1$
8 Let $q = p + 2$, then $pq + 1 = p(p + 2) + 1$
$= p^2 + 2p + 1 = (p + 1)^2$, which is divisible by $p + 1$.
$p + 1 = \frac{2p + 2}{2} = \frac{p + p + 2}{2} = \frac{p + q}{2}$, which is the average of p and q.
9 a $n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$
b Show that if n^2 is even, n is even (9a) and if n is even, n^2 is even (Example 5a).
10 a $2a + 2b + 1 = 2(a + b) + 1$
b Show **10a** and $2a + 1 + 2b + 1 = 2(a + b + 1)$, $2a + 2b = 2(a + b)$
11 $(2a + 1)(2b + 1) = 4ab + 2a + 2b + 1$
$= 2(2ab + a + b) + 1$
12 $2a(2b + 1) = 2(2ab + a)$, $2a \times 2b = 2(2ab)$
13 Proof
14 Proof
15 Proof
16 Proof

7.03

1 a i $\frac{649}{200}$ ii $-\frac{260\,539\,287}{500\,000}$
b i -707.375 ii 62.258
2 C
3 a $51.\overline{27}$ b $18.\overline{384615}$ c $3.\overline{142857}$
4 a $\frac{1}{9}$ b $\frac{7}{12}$ c $\frac{3097}{990}$
d $\frac{1\,714\,284}{999\,999}$ e $\frac{3}{2}$
5 a $0.\overline{9}$ b $\frac{1}{1}$
6 a 6, 16, 18
b $\frac{1}{p}$ where p is a prime has $p - 1$ recurring digits
c $\frac{1}{11} = 0.\overline{09}$
7 a $0.\overline{142857}$, $0.\overline{285714}$, $0.\overline{428571}$, $0.\overline{571428}$, $0.\overline{714285}$, $0.\overline{857142}$
b The recurring digits are rotating.

8 a $0.\overline{076923}$, $76 + 923 = 999$

b $\frac{1}{11} = 0.\overline{09}$, $0 + 9 = 9$,

$\frac{1}{17} = 0.\overline{058\,823\,529\,411\,764\,705}$,

$05882352 + 94117647 = 99\,999\,999$

c $\frac{1}{77} = 0.\overline{012987}$, $12 + 987 = 999$

7.04

1 a $\frac{-2}{3}$ or $\frac{2}{-3}$ b $\frac{375}{1000} = \frac{3}{8}$

c $\frac{5670}{1}$ d $\frac{82}{1000} = \frac{41}{500}$

e $\frac{0}{1}$ f $\frac{11}{4}$

2 D

3 a $\frac{29}{21}$ b $\frac{-9}{10} = \frac{9}{-10}$ c $\frac{10}{21}$

d $\frac{17}{2}$ e $\frac{14}{15}$

4 $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} = \frac{e}{f}$

5 $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd} = \frac{e}{f}$

6 a $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$ b Yes

7 a i $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ ii $\frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

iii $\frac{\frac{1}{2}}{\frac{1}{4}} = 2$

b $\frac{1}{2a} \times \frac{1}{2b} = \frac{1}{2(2ab)}$

8 $\left(\frac{a}{b}\right)^2 - 2\left(\frac{a}{b}\right) + 5$ is rational

9 Proof

10 Proof

11 Many possibilities, like $x^4 - 8x^2 + 15 = 0$, which factorises to $(x^2 - 3)(x^2 - 5) = 0$. It has x-intercepts $\pm\sqrt{3}, \pm\sqrt{5}$, y-intercept 15.
Its roots are $\pm\sqrt{3}, \pm\sqrt{5}$ which are not rational.

INVESTIGATION: INFINITE CONTINUED FRACTIONS FOR IRRATIONAL NUMBERS

a 1.5, 1.4, $1.41\overline{6}$, 1.41379… converging to $\sqrt{2} \approx 1.414\,132\,562\ldots$

b i $0 + \cfrac{1}{99 + \cfrac{1}{1}} = 0.01$, $0 + \cfrac{1}{99 + \cfrac{1}{1 + \cfrac{1}{9}}} = 0.01001$

ii $0 + \cfrac{1}{99 + \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{111 + \cfrac{1}{9 + \cfrac{1}{9 + \cfrac{1}{1 + \cfrac{1}{8}}}}}}}} = 0.010010001$

c $1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4 + \ldots}}}}$, $[1;1,2,1,1,4,1,1,6,1,1,8,\ldots]$,

$3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292}}}}$, $[3;7,15,1,292,1,1,1,2,1,3,1,\ldots]$

d Square roots eventually yield repetitive sequences, but cube roots and higher powers yield random sequences.

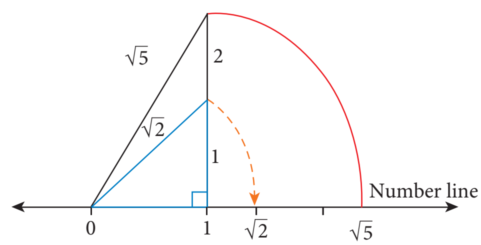
7.05

1 E

2 a $-\sqrt{2} + \sqrt{2} = 0$ b $\sqrt{2} - \sqrt{2} = 0$

c $\sqrt{2} \times \sqrt{8} = 4$ d $\frac{\sqrt{2}}{\sqrt{2}} = 1$

3



4 No, π cannot be written as a fraction.

5 e^{π}, e^2. Other answers are possible.

6 $\sqrt{5} = \frac{a}{b}$, $a^2 = 5b^2$

7 $\log_2(3) = \frac{a}{b}$, $a, b \in Z^+$, $2^{\frac{a}{b}} = 3$, $2^a = 3^b$

8 Solve $a + b + c = 2$, $4a + 2b + c = 5$, $9a + 3b + c = 9$

9 $\frac{1}{10^2} + \frac{1}{10^6} + \frac{1}{10^{11}} + \cdots + \frac{1}{10^{\frac{1}{2}n^2 + \frac{5}{2}n - 1}} + \cdots$

10 $\frac{1}{10^2} + \frac{2}{10^5} + \frac{4}{10^9} + \frac{8}{10^{14}} + \cdots + \frac{2^{n-1}}{10^{\frac{1}{2}n^2 + \frac{3}{2}n}} + \cdots$

11 $b^2 - 4ac > 0$ and $b^2 - 4ac \neq \frac{m^2}{n^2}$, $n \neq 0$, $m, n \in Z$

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12 $x^3 - 2 = 0$

13 a $\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}$

b i $\frac{1+\sqrt{5}}{2}$

ii $\frac{m}{s} = \frac{s}{m-s}$, $\frac{m}{s}$ in simplest form, $\frac{s}{m-s}$ simpler

7.06

1 a i 4 ii $2\sqrt{3}$ iii 1 iv $7+4\sqrt{3}$

b i $\mathbf{Z}^+$ or $\mathbf{N}$

ii $\mathbf{Q}'$

iii $\mathbf{Z}^+$

iv $\mathbf{Q}'$

2 a Yes b Yes c Yes d Yes

3 a $\frac{1}{9}$ b $\frac{1}{2}$ c $-\frac{1}{3}$ d $\frac{1}{6}$

4 Proof

5 Proof

6 Proof

7 a Proof b $a = 3, b = 4, c = 5$

8 $c = -\frac{a^2}{b^2} - \frac{a}{b}$, $b \neq 0$, $a, b \in \mathbf{Z}$ and $c \leq \frac{1}{4}$

9 $e = \sqrt{2}, f = \sqrt{3}, g = \sqrt{6}$

7.07

1–6 Proofs

CHAPTER 7 REVIEW

1 C

2 D

3 B

4 C

5 D

6 a $2k + 1 + 2k + 1 + 2k + 1 = 2(3k + 1) + 1$

b $(2k + 1)^3 = 2(4k^3 + 6k^2 + 3k) + 1$

7 $117 \neq 100$

8 a $2^9 - 1$, 9 is not prime b divisible by 7

9 a $\frac{2}{4} = \frac{1}{2}$ b $12 - 9 = 3$

10 Proof

11 Proof

12 a $\frac{-1\,015\,779}{15\,625}$ b 0.3376

13 a $934.7\overline{14285}$ b $0.\overline{012345679}$

14 $\frac{9463}{2475}$

15 a $\frac{-5}{6}$ b $\frac{7}{8}$ c $\frac{723}{10\,000}$

d $\frac{2400}{1}$ e $\frac{29}{5}$

16 a $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$ b $\frac{7}{0}$ undefined

17 Proof

18 Proof

19 Proof

20 Proof

21 $\frac{1}{10^2} + \frac{1}{10^7} + \frac{1}{10^{15}} + \cdots + \frac{1}{10^{\frac{3n^2}{2}+\frac{n}{2}}} + \cdots$

22 Proof

23 a $2^{\frac{2}{3}}$ b $\sqrt{3} \times \sqrt{3} = 3$

24 $c \leq 0$

25 Proof

26 Proof

27 a i 1, 2, 4, 8, 16, 31, 62, 124, 248, 496

ii 31, prime

iii 62, 2×31

iv 124, 248, 496, $2^n \times 31$

v A perfect number

b i $2^4(2^5 - 1)$ ii 16 iii 31

c $1 + 2 + 4 + \cdots + 2^{p-1} = \frac{1(2^p - 1)}{2 - 1} = 2^p - 1$

d i $(x - 1)(1 + x)$, $(x - 1)(1 + x + x^2)$, $(x - 1)(1 + x + x^2 + x^3)$, $(x - 1)(1 + x + x^2 + x^3 + x^4)$, $(x - 1)(1 + x + x^2 + x^3 + \cdots + x^{n-1})$,

ii If p is not prime then $2^p - 1$ is not prime,

$\left(2^m\right)^n - 1$, $m, n \in \mathbf{Z}^+\backslash\{1\}$, $x = 2^m \geq 4$, $x^n - 1 = (x - 1)(1 + x + x^2 + \cdots + x^{n-1})$,

$x - 1 > 1$, and $(1 + x + x^2 + \cdots + x^{n-1}) > 1$

iii For $p = 11$, $2^{11} - 1 = 23 \times 89$ so it is not prime.

8.01

1 **a, e, f**

2 a $x_{12} = 2$ b $x_{21} = 1$ c $x_{34} = -2$

d $x_{22} = -5$ e $x_{32} = 3$

3 a $r_{31} = r_{25}$ b $2r_{14} = r_{45}$

c $r_{32} = 2r_{15}$ d $r_{42} = -3r_{44}$

e $r_{25} = -4r_{34}$ f $r_{35} = -5r_{34}$

4 a 2×2 square matrix with all elements 1

b 1×3 row matrix with all elements 2

c 2×1 column matrix

d 3×3 diagonal matrix with diagonal elements of 3

e 3×4 matrix with elements of 1 and 2

5 a $\mathbf{M} = \begin{bmatrix} 3 & 2 & 4 & 1 & 5 & 0 & 3 & 6 & 1 \\ 1 & 8 & 1 & 7 & 1 & 8 & 0 & 5 & 8 \\ 4 & 0 & 5 & 6 & 4 & 5 & 9 & 1 & 7 \\ 7 & 9 & 2 & 0 & 1 & 1 & 10 & 9 & 1 \end{bmatrix}$

b 0

c 1

d 0

e 4×9

6 a USD \$1.05664

b $\begin{bmatrix} 1 & 1.05664 & 0.65720 & 94.3892 \\ 0.94642 & 1 & 0.62192 & 89.3362 \\ 1.52148 & 1.60769 & 1 & 143.625 \\ 0.01060 & 0.01120 & 0.00696 & 1 \end{bmatrix}$

c 89.3362 d 1.60769 e $e_{ij} = \frac{1}{e_{ji}}$

7 a 7×6 b 50.7 c 1948

d maximum temperature range = 72.7

8 a $\mathbf{P} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 5 & 2 & 0 & 0 \\ 0 & 6 & 4 & 0 \end{bmatrix}$

b Mosquito larvae do not eat the other animals.

c Water beetles do not eat each other.

9 $\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 \end{bmatrix}$

8.02

1 a $\begin{bmatrix} 6 & 0 \\ -3 & -6 \end{bmatrix}$ b $\begin{bmatrix} \frac{6}{5} & \frac{12}{5} \\ \frac{18}{5} & -\frac{6}{5} \end{bmatrix}$ c $\begin{bmatrix} 15 & 30 \\ 45 & -15 \end{bmatrix}$

d $\begin{bmatrix} -8 & 0 \\ 4 & 8 \end{bmatrix}$ e $\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$

2 a $\begin{bmatrix} -8 & -6 & -2 \\ 0 & -6 & -12 \\ -10 & 0 & 2 \\ 0 & -10 & 4 \end{bmatrix}$ b $\begin{bmatrix} -\frac{1}{2} & -2 & -\frac{3}{2} \\ 1 & -\frac{3}{2} & \frac{1}{2} \\ 2 & 1 & 2 \\ -2 & -\frac{3}{2} & -1 \end{bmatrix}$

c $\begin{bmatrix} 1 & 4 & 3 \\ -2 & 3 & -1 \\ -4 & -2 & -4 \\ 4 & 3 & 2 \end{bmatrix}$ d $\begin{bmatrix} 12 & 9 & 3 \\ 0 & 9 & 18 \\ 15 & 0 & -3 \\ 0 & 15 & -6 \end{bmatrix}$

3 a $\begin{bmatrix} 15 & 0 & -5 & 10 \\ 25 & -20 & 15 & 5 \\ -5 & 10 & 0 & 20 \end{bmatrix}$

b $\begin{bmatrix} -3 & -12 & -9 & 15 \\ 9 & -6 & 3 & 18 \\ 0 & -15 & 6 & -9 \end{bmatrix}$

c $\begin{bmatrix} 12 & 0 & -4 & 8 \\ 20 & -16 & 12 & 4 \\ -4 & 8 & 0 & 16 \end{bmatrix}$

d $\begin{bmatrix} -5 & -20 & -15 & 25 \\ 15 & -10 & 5 & 30 \\ 0 & -25 & 10 & -15 \end{bmatrix}$

4 a $\begin{bmatrix} 4.08 & 8.33 & 14.28 \\ 7.14 & 13.43 & 20.06 \\ 6.46 & 14.28 & 18.70 \\ 10.03 & 20.06 & 24.48 \end{bmatrix}$

b $\begin{bmatrix} 4.49 & 9.16 & 15.71 \\ 7.85 & 14.77 & 22.07 \\ 7.11 & 15.71 & 20.57 \\ 11.03 & 22.07 & 26.93 \end{bmatrix}$

5 a $\begin{bmatrix} 7000 \\ 5600 \\ 8750 \\ 8050 \\ 6650 \end{bmatrix}$ b $\begin{bmatrix} 240 \\ 60 \\ 420 \\ 120 \\ 180 \end{bmatrix}$ c Total sales = $37 070

8.03

1 a $\begin{bmatrix} -4 & 4 \\ 7 & -12 \end{bmatrix}$ b $\begin{bmatrix} 8 & -1 \\ 6 & -2 \end{bmatrix}$ c $\begin{bmatrix} -3 & 10 \\ 17 & -13 \end{bmatrix}$

d $\begin{bmatrix} 13 & 6 \\ 4 & -13 \end{bmatrix}$ e $\begin{bmatrix} 19 & 4 \\ 21 & -7 \end{bmatrix}$

2 a $\begin{bmatrix} 5 & 8 & 9 \\ 0 & 5 & 10 \\ 5 & 6 & 1 \\ 7 & 5 & -2 \end{bmatrix}$ b $\begin{bmatrix} 5 & 8 & 9 \\ 0 & 5 & 10 \\ 5 & 6 & 1 \\ 7 & 5 & -2 \end{bmatrix}$ c $\begin{bmatrix} 0 & 1 & 5 \\ 2 & -1 & 5 \\ 4 & 8 & 6 \\ 3 & -3 & -2 \end{bmatrix}$

d $\begin{bmatrix} 8 & 23 & 33 \\ 0 & 11 & 22 \\ 5 & 24 & 7 \\ 28 & 5 & -2 \end{bmatrix}$ e $\begin{bmatrix} 12 & 11 & 13 \\ 4 & 7 & 28 \\ 23 & 16 & 9 \\ 6 & 9 & -10 \end{bmatrix}$

3 a $\begin{bmatrix} 4 & 4 & 2 & -3 \\ 2 & -2 & 2 & -5 \\ -1 & 7 & -2 & 7 \end{bmatrix}$

b $\begin{bmatrix} 7 & -1 & 7 & -8 \\ -4 & 10 & -10 & -4 \\ 4 & 12 & -4 & 8 \end{bmatrix}$

c $\begin{bmatrix} 30 & -15 & 8 & -1 \\ 17 & 8 & -15 & 10 \\ 8 & 29 & -6 & 31 \end{bmatrix}$

d $\begin{bmatrix} 37 & -21 & 21 & -16 \\ 2 & 34 & -40 & 6 \\ 18 & 44 & -12 & 38 \end{bmatrix}$

e $\begin{bmatrix} -3 & -39 & -1 & 11 \\ -10 & 32 & -36 & 37 \\ 17 & -19 & 6 & -23 \end{bmatrix}$

4 a $\begin{bmatrix} -8 & 4 \\ 9 & -8 \end{bmatrix}$ b $\begin{bmatrix} -2 & 13 \\ 12 & -4 \end{bmatrix}$ c $\begin{bmatrix} -9 & -2 \\ -1 & -7 \end{bmatrix}$

d $\begin{bmatrix} 7 & -6 \\ -14 & -7 \end{bmatrix}$ e $\begin{bmatrix} -1 & 32 \\ 33 & -11 \end{bmatrix}$

5 a $\begin{bmatrix} -3 & 2 & 7 \\ 0 & -1 & -2 \\ -5 & 6 & 3 \\ 7 & -5 & 2 \end{bmatrix}$ b $\begin{bmatrix} 3 & -2 & -7 \\ 0 & 1 & 2 \\ 5 & -6 & -3 \\ -7 & 5 & -2 \end{bmatrix}$

c $\begin{bmatrix} -2 & -9 & -11 \\ 2 & -5 & -3 \\ 4 & -4 & 2 \\ -11 & -3 & -2 \end{bmatrix}$ d $\begin{bmatrix} 0 & 17 & 31 \\ 0 & 5 & 10 \\ -5 & 24 & 9 \\ 28 & -5 & 2 \end{bmatrix}$

e $\begin{bmatrix} -12 & -7 & 7 \\ 4 & -11 & -8 \\ -7 & 16 & 15 \\ 6 & -21 & 2 \end{bmatrix}$

6 a $\begin{bmatrix} 2 & -4 & -4 & 7 \\ 8 & -6 & 4 & 7 \\ -1 & -3 & 2 & 1 \end{bmatrix}$ b $\begin{bmatrix} 5 & -9 & 1 & 2 \\ 2 & 6 & -8 & 8 \\ 4 & 2 & 0 & 2 \end{bmatrix}$

c $\begin{bmatrix} -6 & 15 & -16 & 17 \\ 23 & -40 & 39 & -2 \\ -16 & -13 & 6 & 1 \end{bmatrix}$ d $\begin{bmatrix} 5 & 25 & 9 & -13 \\ -4 & -6 & 10 & -30 \\ -6 & 22 & -8 & 18 \end{bmatrix}$

e $\begin{bmatrix} -3 & -39 & -1 & 11 \\ -10 & -8 & -38 & 37 \\ 17 & -19 & 6 & -23 \end{bmatrix}$

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7 a $\begin{bmatrix} 300 & 400 & 200 & 150 \\ 400 & 220 & 300 & 200 \\ 250 & 150 & 400 & 150 \end{bmatrix}$

b $\begin{bmatrix} 30 & 70 & 20 & 20 \\ 20 & 30 & 40 & 30 \\ 40 & 10 & 50 & 10 \end{bmatrix}$

c $\begin{bmatrix} 50 & 100 & 40 & 15 \\ 15 & 50 & 30 & 10 \\ 30 & 25 & 20 & 40 \end{bmatrix}$

d $\begin{bmatrix} 320 & 430 & 220 & 145 \\ 395 & 240 & 290 & 180 \\ 240 & 165 & 370 & 180 \end{bmatrix}$

8 a **F** + **M** + **L** b $\begin{bmatrix} 10 & 1 & 4 \\ 6 & 0 & 9 \\ 4 & 3 & 8 \\ 8 & 2 & 5 \\ 4 & 2 & 9 \\ 9 & 0 & 6 \end{bmatrix}$

c Team 1 (1st team)

8.04

1 a $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$ b $\begin{bmatrix} -7 & 4 \end{bmatrix}$ c $\begin{bmatrix} 6 & -2 & 4 \\ -9 & 3 & -6 \\ 12 & -4 & 8 \end{bmatrix}$

d $\begin{bmatrix} -2 & 1 \\ 6 & 3 \end{bmatrix}$ e $\begin{bmatrix} -16 \\ 6 \end{bmatrix}$

2 a $\begin{bmatrix} 4 & 8 & -11 \end{bmatrix}$ b $\begin{bmatrix} -7 \\ -2 \\ -10 \end{bmatrix}$

c $\begin{bmatrix} 1 & 13 \\ -2 & 14 \end{bmatrix}$ d $\begin{bmatrix} -1 & -13 \\ 2 & -14 \end{bmatrix}$

e $\begin{bmatrix} -11 & 12 & 12 \\ 18 & -6 & 16 \\ 7 & -9 & 6 \end{bmatrix}$

3 Both $\begin{bmatrix} 33 & 21 \\ 35 & 40 \end{bmatrix}$, not usually expected

4 Both $\begin{bmatrix} -11 & 7 & -11 \\ 7 & 13 & 7 \\ -11 & 7 & -11 \end{bmatrix}$, not usually expected

5 Proof

6 Proof

7 a $\begin{bmatrix} 12 & 10 & 8 \\ 15 & 8 & 4 \\ 10 & 10 & 10 \\ 12 & 12 & 4 \end{bmatrix}$

b $\begin{bmatrix} 117.80 \\ 155.00 \\ 190.00 \end{bmatrix}$

c $\begin{bmatrix} 4483.60 \\ 3767.00 \\ 4628.00 \\ 4033.60 \end{bmatrix}$

8 a Diagonal zeros mean that there is no trucking from a city to the same city.

b $\mathbf{H} = \begin{bmatrix} 0 & 4 & 4 & 4 & 4 \\ 4 & 0 & 4 & 4 & 4 \\ 4 & 4 & 0 & 4 & 4 \\ 4 & 4 & 4 & 0 & 4 \\ 4 & 4 & 4 & 4 & 0 \end{bmatrix}$

$\mathbf{T} = \begin{bmatrix} 0 & 14.80 & 13.80 & 10.40 & 9.00 \\ 14.80 & 0 & 11.20 & 20.40 & 19.40 \\ 13.80 & 11.20 & 0 & 20.20 & 18.40 \\ 10.40 & 20.40 & 20.20 & 0 & 15.00 \\ 9.00 & 19.40 & 18.40 & 15.00 & 0 \end{bmatrix}$

c Delivery for medium parcel = 1.5**C** + **H**

d Delivery for large parcel = 3**C** + 2**H**

9 a $\mathbf{P} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

b $\mathbf{FP} = \begin{bmatrix} 12 \\ 9 \\ 4 \\ 7 \\ 3 \\ 9 \end{bmatrix}$, $\mathbf{MP} = \begin{bmatrix} 10 \\ 3 \\ 5 \\ 9 \\ 7 \\ 9 \end{bmatrix}$, $\mathbf{LP} = \begin{bmatrix} 9 \\ 6 \\ 6 \\ 10 \\ 4 \\ 9 \end{bmatrix}$

c Both give $\begin{bmatrix} 31 \\ 18 \\ 15 \\ 26 \\ 14 \\ 27 \end{bmatrix}$. Team 1 wins with 31 points, also it shows that matrix multiplication is distributive over addition.

8.05

1 a $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$ b $\begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{1}{10} & \frac{3}{10} \end{bmatrix}$ c $\begin{bmatrix} 0 & -1 \\ -\frac{1}{3} & -1 \end{bmatrix}$

2 a $\begin{bmatrix} \frac{1}{2} & -\frac{2}{3} \\ -\frac{1}{2} & 1 \end{bmatrix}$ b $\begin{bmatrix} \frac{1}{15} & \frac{4}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{bmatrix}$

c Does not exist.

3 a nilpotent b no c nilpotent

4 a idempotent b no c idempotent

5 a 34 b 58 c −12

d −42 e 21

6 a 53 b −175 c 165

d 112 e 43

7 a $\begin{bmatrix} 0 & \frac{1}{2} \\ -1 & \frac{1}{2} \end{bmatrix}$ b $\begin{bmatrix} -\frac{1}{5} & -\frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$ c $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

d $\begin{bmatrix} 2 & 1 \\ -1\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$ e Does not exist.

8 $\mathbf{B}^{-1} = \begin{bmatrix} 15 & 4 & -5 \\ -12 & -3 & 4 \\ -4 & -1 & 1 \end{bmatrix}$

9 $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

10–12 Proofs

13 a $\mathbf{P} = \begin{bmatrix} 25 & 20 & 50 & 25 & 30 & 50 & 50 \\ 35 & 35 & 80 & 35 & 40 & 70 & 75 \\ 45 & 40 & 100 & 40 & 40 & 90 & 85 \end{bmatrix}$

b $\mathbf{A} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

c $\mathbf{PA} = \begin{bmatrix} 25 & 20 & 50 & 25 & 30 & 50 & 50 \\ 35 & 35 & 80 & 35 & 40 & 70 & 75 \\ 45 & 40 & 100 & 40 & 40 & 90 & 85 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 250 \\ 370 \\ 440 \end{bmatrix}$

d $\mathbf{B} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

e $\mathbf{BP} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \times$

$\begin{bmatrix} 25 & 20 & 50 & 25 & 30 & 50 & 50 \\ 35 & 35 & 80 & 35 & 40 & 70 & 75 \\ 45 & 40 & 100 & 40 & 40 & 90 & 85 \end{bmatrix}$

$\mathbf{BP} = \begin{bmatrix} 105 & 95 & 230 & 100 & 110 & 210 & 210 \end{bmatrix}$

f $\mathbf{C} = \begin{bmatrix} 7.50 & 11.00 & 15.00 \end{bmatrix}$

g Income = $\mathbf{C} \times \mathbf{PA}$

$= \begin{bmatrix} 7.50 & 11.00 & 15.00 \end{bmatrix} \times \begin{bmatrix} 250 \\ 370 \\ 440 \end{bmatrix}$

= \$12 545

8.06

1 a $(\mathbf{M} + 4\mathbf{I})\mathbf{M}$ b $(7\mathbf{G} - \mathbf{I})\mathbf{X}$ c $\mathbf{A}(\mathbf{B} - 3\mathbf{I})$

d $\mathbf{H}(\mathbf{HG} + 4\mathbf{G})\mathbf{G}$ e no factors

2 a $\begin{bmatrix} 19 & 37 \\ -13 & -26 \end{bmatrix}$ b $\begin{bmatrix} \frac{1}{2} & -1\frac{1}{2} \\ 0 & -1 \end{bmatrix}$ c $\begin{bmatrix} 7 \\ 1 \end{bmatrix}$

d $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ e $\begin{bmatrix} -\frac{13}{25} & \frac{14}{25} \\ \frac{54}{25} & -\frac{37}{25} \end{bmatrix}$

3 a $\begin{bmatrix} 52 & 76 \\ -22 & -32 \end{bmatrix}$ b $\begin{bmatrix} 3 & 4 \\ -5 & -2 \end{bmatrix}$ c $\begin{bmatrix} -\frac{1}{2} \\ 0 \end{bmatrix}$

d $\begin{bmatrix} 4\frac{4}{5} & -4\frac{3}{5} \\ 3\frac{2}{5} & -3\frac{4}{5} \end{bmatrix}$ e $\begin{bmatrix} 0 & -1\frac{1}{5} & 4\frac{1}{5} \\ 1 & -4 & 13 \end{bmatrix}$

4 a $\begin{bmatrix} -8 & 0 \\ 22 & 3 \end{bmatrix}$ b $\begin{bmatrix} -12 & 9 \\ 16 & -13 \end{bmatrix}$ c $\begin{bmatrix} 1.6 & -1.8 \\ -8.6 & -1.2 \end{bmatrix}$

d $\begin{bmatrix} -2 & -2\frac{1}{2} \\ -1 & -3 \end{bmatrix}$ e $\begin{bmatrix} -0.65 & -4.7 & 7.9 \\ 0.15 & 7.7 & -9.9 \\ -0.65 & 12.3 & -17.1 \end{bmatrix}$

CHAPTER 8 REVIEW

1 B

2 E

3 A

4 C

5 B

6 $\begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$

7 $\begin{bmatrix} 4 & 5 & 4 & 5 \\ -8 & -7 & -8 & -7 \end{bmatrix}$

8 $\begin{bmatrix} -1 & 1 \\ -2\frac{1}{2} & 3 \end{bmatrix}$

9 a $\begin{vmatrix} 3 & 3 \\ 8 & 9 \end{vmatrix} = 3, \begin{bmatrix} 3 & 3 \\ 8 & 9 \end{bmatrix}^{-1} = \begin{bmatrix} 3 & -1 \\ -2\frac{2}{3} & 1 \end{bmatrix}$

b $\begin{vmatrix} -4 & 10 \\ -2 & 5 \end{vmatrix} = 0$, so no inverse exists.

10 $\mathbf{P}(\mathbf{P} + \mathbf{Q})\mathbf{Q}$

11 $\begin{bmatrix} \frac{1}{2} & -3 \\ -\frac{1}{4} & 7 \end{bmatrix}$

12 $\begin{bmatrix} 10 & -4\frac{1}{2} \\ -21\frac{1}{2} & 7\frac{1}{4} \end{bmatrix}$

13 Proof

14 a $\begin{bmatrix} 12.5 & 13.2 & 15.0 & 16.4 & 17.5 & 18.0 \end{bmatrix}$

b $\begin{bmatrix} 3 & 2 & 4 \\ 1 & 2 & 5 \\ 2 & 3 & 0 \\ 3 & 1 & 1 \\ 1 & 3 & 0 \\ 0 & 2 & 2 \end{bmatrix}$

c George = \$147.40, Frank = \$201.30 and Maria = \$168.40

9.01

1 A

2 D

3 D

4 D

5 C

6 a -1 b $-\sqrt{2}$ c $-\frac{2}{\sqrt{3}}$ d 2

e $\sqrt{3}$ f $-\sqrt{2}$ g $\sqrt{2}$ h $\sqrt{3}$

7 a $\sec(\theta)$ b $-\sec(\theta)$ c $-\mathrm{cosec}(\theta)$

d $\cot(\theta)$ e $-\mathrm{cosec}(\theta)$ f $-\cot(\theta)$

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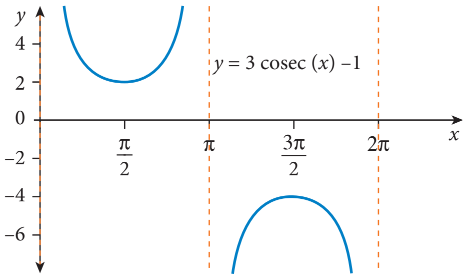
8



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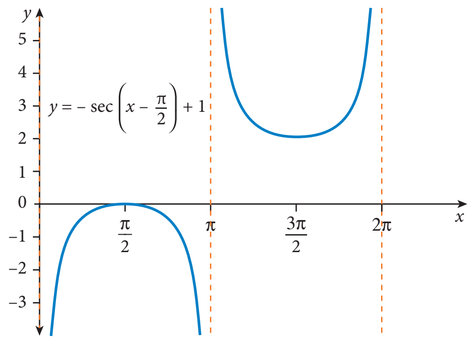

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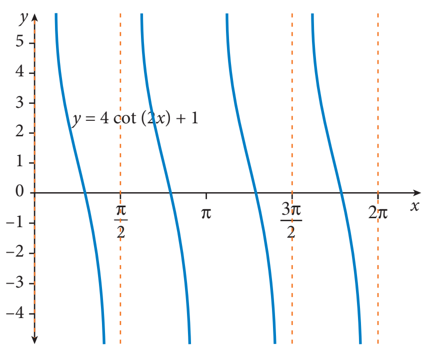

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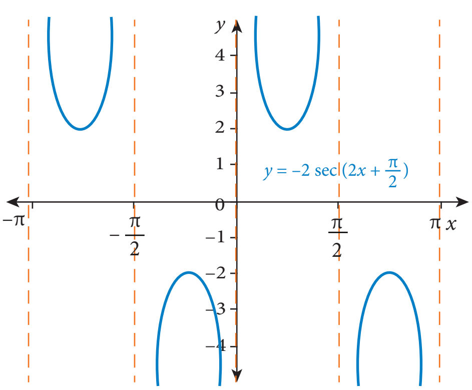


12 $\sec\left(\frac{\pi}{4}\right)=\sqrt{2}$ and $\text{cosec}\left(\frac{\pi}{4}\right)=\sqrt{2}$. These expressions are equal. Note that $\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2}$.

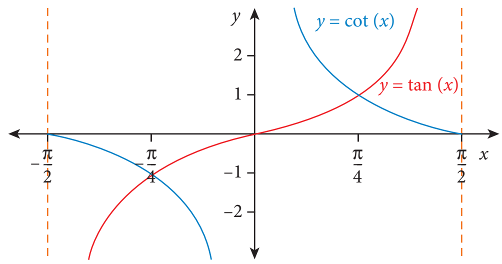
13 $\sec\left(\frac{\pi}{3}\right)=2$ and $\text{cosec}\left(\frac{\pi}{6}\right)=2$. These expressions are equal. Note that $\frac{\pi}{3}+\frac{\pi}{6}=\frac{\pi}{2}$.

14 a $\frac{2\sqrt{3}}{3}$ b 1 c $-\frac{2\sqrt{3}}{3}$ d $\frac{2\sqrt{3}}{3}$

e 1 f not defined.

g $-\frac{\sqrt{3}}{3}$ h $\sqrt{2}$ i −1 j 2

15



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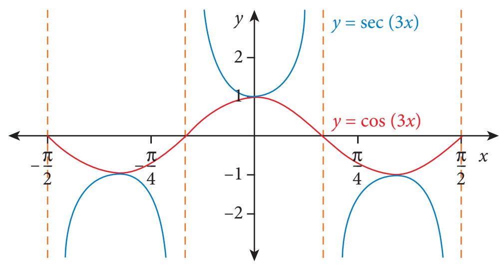

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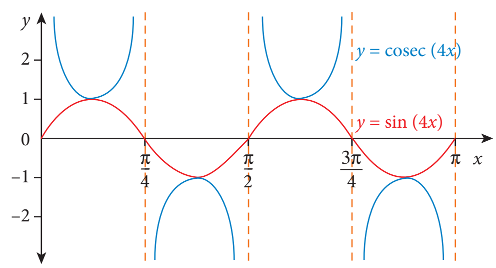

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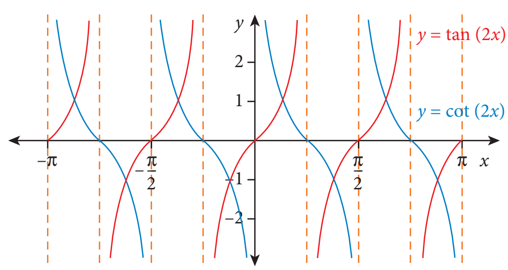

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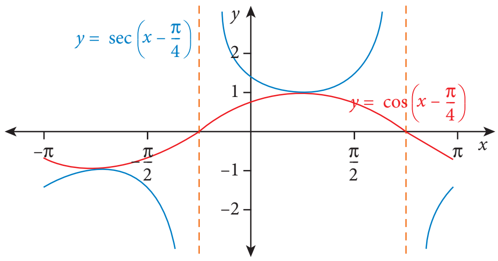

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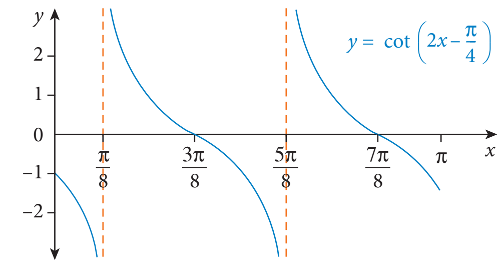

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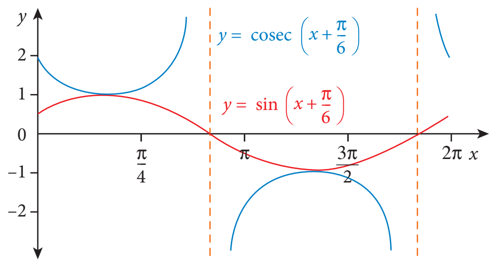

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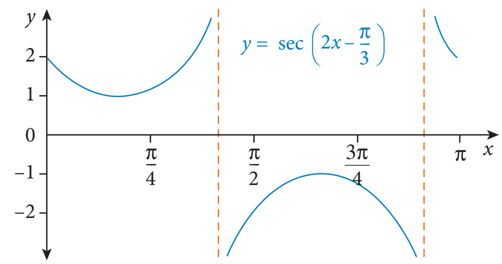

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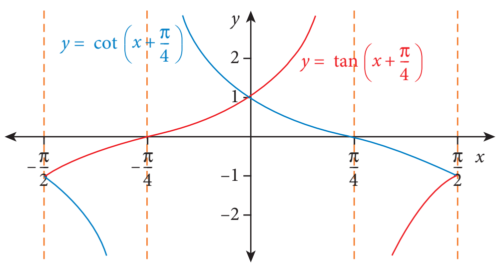

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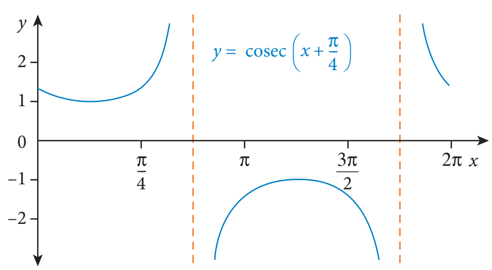

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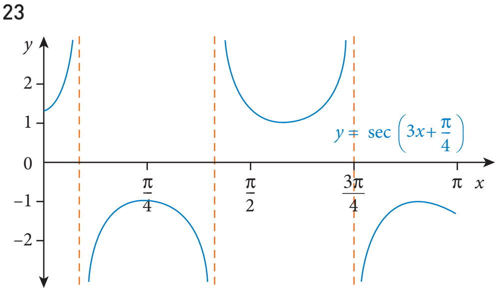

9.02

1–12 Proofs

9.03

1 **a** $\sin(2x)\cos(y) + \cos(2x)\sin(y)$

b $\cos(4x)\cos(3y) + \sin(4x)\sin(3y)$

c $\sin(3p)\cos(5q) - \cos(3p)\sin(5q)$

d $\dfrac{\tan(2e) - \tan(f)}{1 + \tan(2e)\tan(f)}$

e $\cos(3\alpha)\cos(2\beta) - \sin(3\alpha)\sin(2\beta)$

f $\sin(4m)\cos(n) + \cos(4m)\sin(n)$

g $\dfrac{\tan(w) + \tan(3x)}{1 - \tan(w)\tan(3x)}$

h $\sin(2D)\cos(5E) - \cos(2D)\sin(5E)$

i $\cos(P)\cos(4Q) + \sin(P)\sin(4Q)$

2 **a** $\sin(x)$ **b** $-\cos(x)$ **c** $\cot(x)$

d $\sec(x)$ **e** $\tan(x)$ **f** $-\text{cosec}(x)$

g $-\sin(x)$ **h** $-\sin(x)$ **i** $\tan(x)$

3–12 Proofs

9.04

1–5 Proofs

6 **a** $\sin(4x) = 2\sin(2x)\cos(2x)$

b $\cos(6A) = 2\cos^2(3A) - 1$

c $\tan(4y) = \dfrac{2\tan(2y)}{1 - \tan^2(2y)}$

d $\cos(y) = \cos^2\left(\dfrac{y}{2}\right) - \sin^2\left(\dfrac{y}{2}\right)$

e $\tan(4x) = \dfrac{2\tan(2x)}{1 - \tan^2(2x)}$

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7 a $4\cos^3(x)\sin(x) - 4\cos(x)\sin^3(x)$

b $\cos^6(A) - \sin^6(A) - 15\cos^4(A)\sin^2(A) + 15\cos^2(A)\sin^4(A)$

c $\dfrac{4\tan(y) - 4\tan^3(y)}{\tan^4(y) - 6\tan^2(y) + 1}$

(Other answers are possible.)

8–14 Proofs

9.05

1 a $2-\sqrt{3}$ b $\dfrac{\sqrt{2-\sqrt{3}}}{2} = \dfrac{\sqrt{6}-\sqrt{2}}{4}$ c $-(2+\sqrt{3})$

d $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ e $-(2+\sqrt{3})$ f $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

g $-\dfrac{\sqrt{6}+\sqrt{2}}{4}$ h $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ i $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

2 $2+\sqrt{3}$

3 a $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ b $\dfrac{\sqrt{\sqrt{2}+2}}{2}$ c $\sqrt{3}+2$

d $\dfrac{\sqrt{2-\sqrt{2}}}{2}$ e $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ f $\sqrt{2}-1$

9.06

1 a 0.633 b 0.133 c 0.112 d 0.754

2 a $\sin(12x) - \sin(2x)$

b $4\cos(6x) - 4\cos(12x)$

c $3\cos(10x) + 3\cos(2x)$

d $2\sin(11x) + 2\sin(x)$

e $\frac{1}{2}\sin(16x) - \frac{1}{2}\sin(4x)$

f $1\frac{1}{2}\cos(24x) + 1\frac{1}{2}\cos(4x)$

g $2\frac{1}{2}\cos(x) - 2\frac{1}{2}\cos(5x)$

h $3\frac{1}{2}\cos(20x) + 3\frac{1}{2}\cos(2x)$

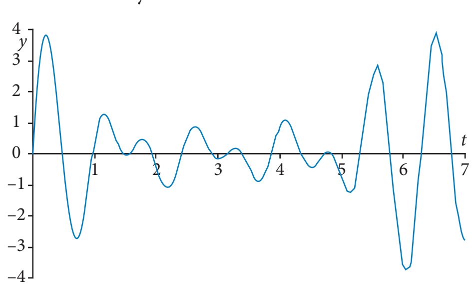
i $\frac{1}{2}\sin(9x) - \frac{1}{2}\sin(7x)$

3 a $\dfrac{\sqrt{3}+1}{4}$ b $\dfrac{\sqrt{3}+1}{4}$

c $\dfrac{\sqrt{3}-1}{4}$ d $\dfrac{\sqrt{3}-1}{2}$

4–9 Proofs

10 Write as $4\cos(x)\sin(6\frac{1}{2}x)\cos\left(\frac{1}{2}x\right)$. Graph shows the sizes clearly.



9.07

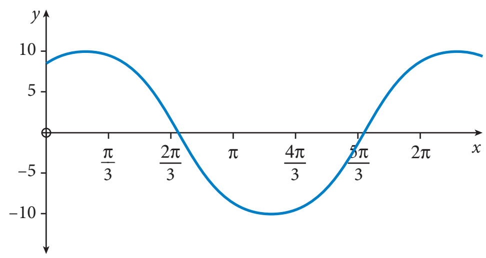
1 a i $10\sin(x+\theta)$, where $\theta = \arctan\left(\frac{4}{3}\right)$

ii $10\cos(x+\theta)$, where $\theta = -\arctan\left(\frac{3}{4}\right)$ or $2\pi - \arctan\left(\frac{3}{4}\right)$

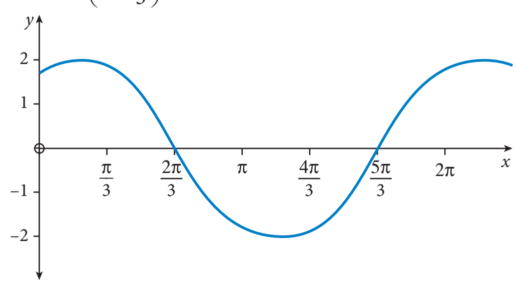
iii $10\sin(x-\theta)$, where $\theta = -\arctan\left(\frac{4}{3}\right)$ or $2\pi - \arctan\left(\frac{4}{3}\right)$

iv $10\cos(x-\theta)$, where $\theta = \arctan\left(\frac{3}{4}\right)$

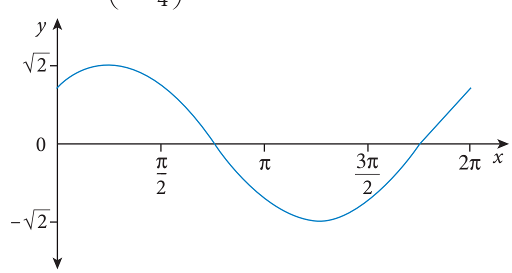
b The graphs are identical as:



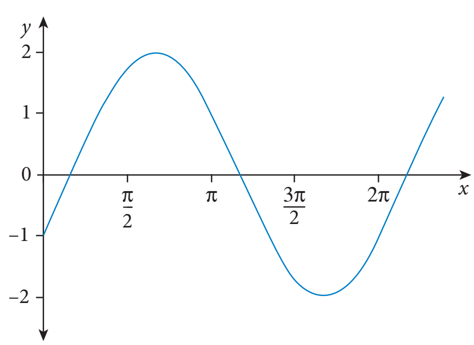
2 $2\sin\left(x+\frac{\pi}{3}\right)$



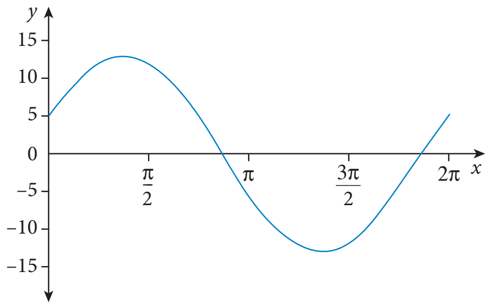
3 $\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)$



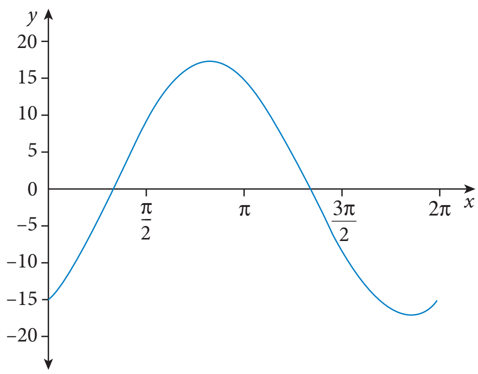
4 $2\cos\left(x+\frac{4\pi}{3}\right)$



5 $13\sin(x+\theta)$, where $\theta = \arctan\left(\frac{5}{12}\right)$



6 $17\sin(x-\theta)$, where $\theta = \arctan\left(\frac{15}{8}\right)$



7 a $2\sin(5x)\cos(2x)$
b $2\cos(5x)\sin(3x)$
c $2\cos(7x)\cos(2x)$
d $-2\sin(9x)\sin(2x)$
e $2\sin\left(6\frac{1}{2}x\right)\cos\left(2\frac{1}{2}x\right)$
f $-2\cos\left(3\frac{1}{2}x\right)\sin\left(1\frac{1}{2}x\right)$
g $2\cos(3x)\cos(2x)$
h $2\sin(7x)\sin(4x)$
i $2\sin\left(1\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)$

8 0

9 $\frac{\sqrt{6}}{2}$

10 $\sqrt{2}$

11–14 Proofs

15 a $2\cos\left(x+\frac{1}{2}h\right)\sin\left(\frac{1}{2}h\right)$
b $\cos\left(x+\frac{1}{2}h\right)\dfrac{\sin\left(\frac{1}{2}h\right)}{\frac{1}{2}h}$
c $\cos(x)$, with $z = \frac{1}{2}h$

9.08

1–4 Proofs

5 $-\frac{4}{3}\sin^3(x) - 2\sin^2(x) + \sin(x) + 1$

6 $4\cos^4(x) - 6\cos^2(x) + 2$

7–10 Proofs

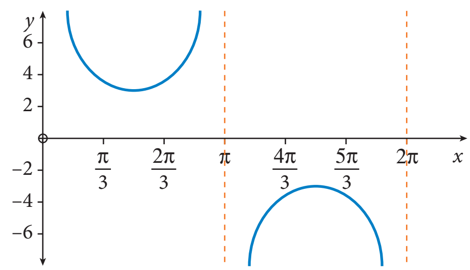
CHAPTER 9 REVIEW

1 C
2 A
3 E
4 C
5 A
6 E
7 E
8 B

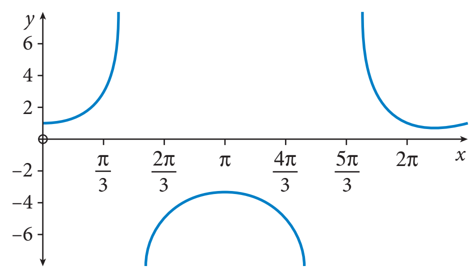
9 a $\sec(\theta)$ b $\operatorname{cosec}(\theta)$
c $-\cot(\theta)$ d $-\sec(\theta)$
e $\cot(\theta)$ f $-\operatorname{cosec}(\theta)$

10 a $\operatorname{cosec}(\theta)$ b $-\sec(\theta)$
c $\tan(\theta)$ d $\operatorname{cosec}(\theta)$
e $\tan(\theta)$ f $\sec(\theta)$

11 $y = 3\operatorname{cosec}(x)$ for $0 \le x \le 2\pi$



12 $y = 2\sec(x) - 1$ for $0 \le x \le 2\pi$



13 Proof

14 $\frac{\sqrt{6}+\sqrt{2}}{4}$

15 $2-\sqrt{3}$

16 $\frac{\sqrt{6}+\sqrt{2}}{4}$

17 $4\sin(x)\cos^3(x) - 4\sin^3(x)\cos(x)$, or equivalent

18–21 Proofs

22 $-6\cos^4(x) + 6\cos^2(x) + 2\cos^3(x) - \cos(x)$

MIXED REVISION 3

Multiple choice

1 A
2 C
3 D

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4 D
5 C
6 C
7 B
8 E
9 A

Short answer

1 a LHS is even, but the RHS is odd.
b (5, 5) or (10, 2)

2 $\begin{vmatrix} 12 & 28 \\ 15 & 35 \end{vmatrix} = 0$, so $\begin{bmatrix} 12 & 28 \\ 15 & 35 \end{bmatrix}$ has no inverse.

3 $\sin(x)\sin(2x)$
$= \sin(x) \times 2\sin(x)\cos(x)$
$= 2\sin^2(x)\cos(x)$
$= 2[1 - \cos^2(x)]\cos(x)$
$= 2\cos(x) - 2\cos^3(x)$

4 $\frac{1}{2\times10} + \frac{1}{2\times10^4} + \frac{1}{2\times10^8} + \dots + \frac{1}{2\times10^{\frac{1}{2}n^2+\frac{3}{2}n-1}} + \dots,$
$n \in \boldsymbol{N}$

5 $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

6 a $\cos(x + 2x)$
$= \cos(x)\cos(2x) - \sin(x)\sin(2x)$
$= \cos(x)[2\cos^2(x) - 1] - \sin(x) \times 2\sin(x)\cos(x)$
$= 2\cos^3(x) - \cos(x) - 2\sin^2(x)\cos(x)$
(Alternate answers are possible)
b $2\cos^3(x) - \cos(x) - 2\sin^2(x)\cos(x)$
$= \cos(x)[2\cos^2(x) - 1 - 2\sin^2(x)]$
$= \cos(x)[2\{1 - 2\sin^2(x)\} - 1 - 2\sin^2(x)]$
$= \cos(x)[2 - 2\sin^2(x) - 1 - 2\sin^2(x)]$
$= \cos(x)[1 - 4\sin^2(x)]$

Application

1 Proof
2 a $2n + 7 = 11, n = 2$
b $2(2k) + 7 = 4k + 7, 2(2k + 1) + 7 = 4k + 9$
c Proof
d $n = 1, n = 7, n = 49$

3 $\begin{bmatrix} -4 & 3 \\ 7 & -5 \end{bmatrix}$

4 $\boldsymbol{X} = \begin{bmatrix} -8 & 0 \\ 22 & 3 \end{bmatrix}$

5 Proof
6 Proof

10.01

1 a $5i$ b $6i$ c $2i\sqrt{5}$ d $\frac{2i\sqrt{2}}{3}$
2 a $\pm i$ b $\pm 3i$ c $\pm 7i$ d $\pm\frac{i}{2}$
3 a -1 b $-i$ c 1 d i
4 a 1 b -1 c $-i$ d -45
5 a 2 b $-6 + 15i$
6 $1 + i$
7 $(x - 4i)(x + 4i)$
8 a 0 b $1 + i$
9 a $-i$ b i c 1
d $-i$ e -1
10 a $(x - 2i)(x + 2i)$ b $(x - 9i)(x + 9i)$

10.02

1 a complex b purely imaginary
c real d real
e purely imaginary f real
g complex h real

2 a $\text{Re}(z) = -2, \text{Im}(z) = -4$
b $\text{Re}(z) = \frac{7}{4}, \text{Im}(z) = \frac{3}{4}$
c $\text{Re}(z) = -6, \text{Im}(z) = \sqrt{2}$
d $\text{Re}(z) = \frac{x^2}{x^2+y^2}, \text{Im}(z) = \frac{-y^2}{x^2+y^2}$

3 a $w = 5 - 4i$ b $w = \sqrt{2} + i\sqrt{7}$
c $w = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$ d $w = -2\sqrt{2} - 5i\sqrt{2}$
e $w = \frac{y + xi}{x^2 + y^2}$

4 a $x = \pm 7i$ b $z = \pm i$ c $w = \pm 5i$
d $x = -2 \pm i$ e $z = \frac{1 \pm i\sqrt{7}}{2}$ f $w = 1 \pm i$
g $x = 1 \pm i\sqrt{3}$ h $z = 3 \pm i$ i $w = 2 \pm 2i$

5 a $\text{Re}(z) = x + 3, \text{Im}(z) = y - 2$
b $\text{Re}(z) = x - 3y, \text{Im}(z) = 2x + y$
c $\text{Re}(z) = -x + 4, \text{Im}(z) = 5 - y$
d $\text{Re}(z) = 4x - y, \text{Im}(z) = -(3x - 2y)$
e $\text{Re}(z) = 1, \text{Im}(z) = \frac{2xy}{x^2 + y^2}$

6 a $z = \frac{-3 \pm i\sqrt{23}}{4}$ b $z = \frac{(-3 \pm \sqrt{3})i}{3}$
c $z = a \pm bi$

10.03

1 a $\bar{z} = -1 + i$ b $\bar{z} = \frac{3 - i}{2}$
c $\bar{z} = -i\sqrt{3} - 1$ d $x - y - (2x - 3y)i$
e $\frac{x^2 - iy^2}{x^2 + y^2}$

2 a 5 b $7\frac{1}{4}$ c 14

3 $(x + 1 - i\sqrt{2})(x + 1 + i\sqrt{2})$

4 a $(x + 2 - i)(x + 2 + i)$ b $(x - 3 + 2i)(x - 3 - 2i)$
c $(x - 1 - i)(x - 1 + i)$

5 a $x = \frac{-1 \pm i\sqrt{15}}{2}$ b $x = \frac{3 \pm i\sqrt{3}}{2}$
c $x = -3 \pm i$ d $x = \frac{-1 \pm i\sqrt{31}}{2}$

6 a $x^2 + 2x + 2 = 0$ b $x^2 - 2x + 10 = 0$
c $x^2 - 2\sqrt{3}x + 4 = 0$

7 a $x = \frac{-i \pm i\sqrt{5}}{2}$ b $x = i$ c $x = i, -2i$

8 a *… if complex, must be complex conjugate pairs*
b *…are not complex conjugate pairs*

9 a Proof b Proof

10.04

1 a $a = 7, b = -4$ b $a = 9, b = 6$
c $a = 1, b = -1$

2 a $3 - 2i$ b $3 + i$
c $21 + 25i$ d $2 - 24i$

3 a $13 + 10i$ b $-3 + 5i$
c $-5 + 12i$ d $16 + 11i$
e $16 - 11i$ f $48 + 14i$
g $21 + 20i$ h $4 + 4i$

4 a $\frac{1}{2} - \frac{1}{2}i$ b $\frac{3}{5} - \frac{4}{5}i$ c $-\frac{6}{13} + \frac{17}{13}i$
d $\frac{1}{17} - \frac{4}{17}i$ e $-\frac{1}{5} - \frac{2}{5}i$

5 a $-\frac{1}{5} - \frac{2}{5}i$ b $-\frac{8}{13} + \frac{i}{13}$ c $-\frac{9}{25} + \frac{12}{25}i$
d $\frac{3}{13} - \frac{2}{13}i$ e -1

6 a $18 - 4i$ b $25 + 28i$
c $\frac{7}{5} + \frac{4}{5}i$ d $-\frac{9}{25} + \frac{13}{25}i$

7 $X = \frac{8}{25}, Y = -\frac{31}{25}$

8 a $\frac{x}{x^2+y^2} - \frac{y}{x^2+y^2}i$ b $\frac{x}{x^2+y^2} + \frac{y}{x^2+y^2}i$
c $\frac{x^2-y^2}{(x^2+y^2)^2} - \frac{2xy}{(x^2+y^2)^2}i$
d $-\frac{y}{x^2+y^2} - \frac{x}{x^2+y^2}i$
e $\frac{x^2+y^2-1}{x^2+(y+1)^2} - \frac{2x}{x^2+(y+1)^2}i$

9 Substitute $z = 1 + i$ into $z^2 - 2z + 2 = 0$.

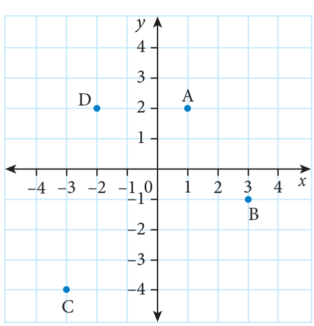
10 Substitute each solution in the LHS.

11 Substitute each solution in the LHS.

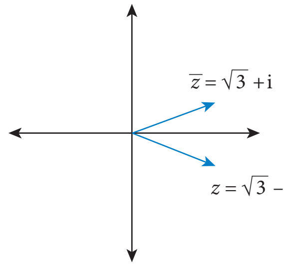
12 a, b Expand the LHS.

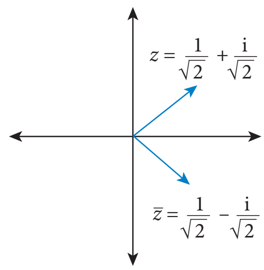
10.05

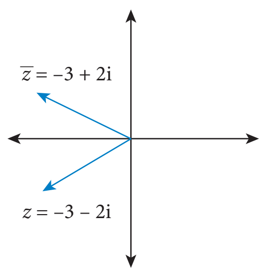
1 i a $A(1, 2)$ b $B(3, -1)$
c $C(-3, -4)$ d $D(-2, 2)$

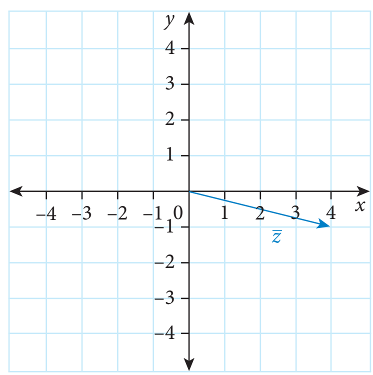


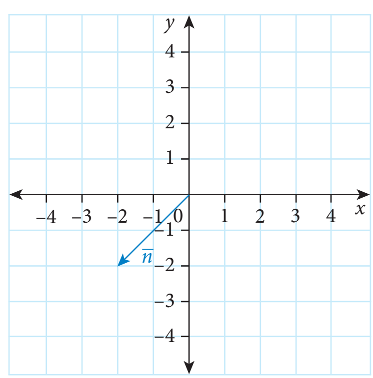
2 $z = 3 + i, u = -3 + 3i, v = -4 - 4i, w = 2 - 4i$

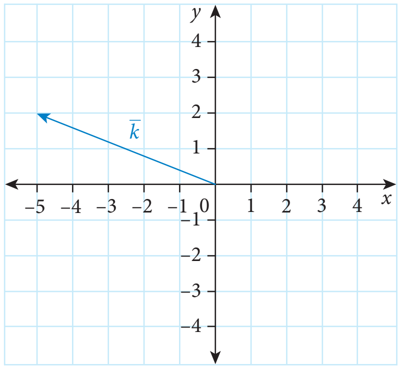
3 a


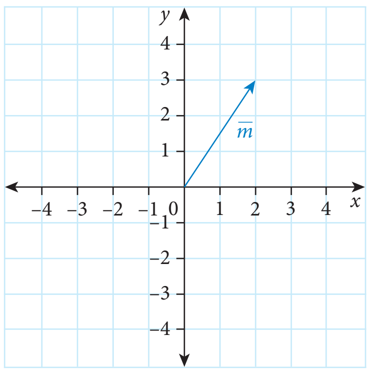
b


c


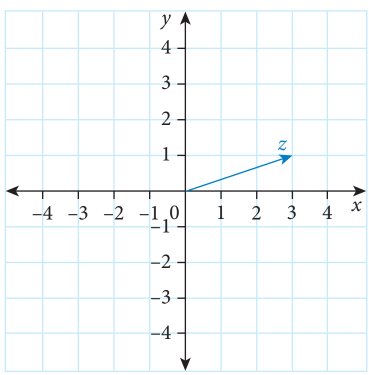
4 a $z = 4 + i, n = -2 + 2i, k = -5 - 2i, m = 2 - 3i$
b $\bar{z} = 4 - i, \bar{n} = -2 - 2i, \bar{k} = -5 + 2i, \bar{m} = 2 + 3i$
c


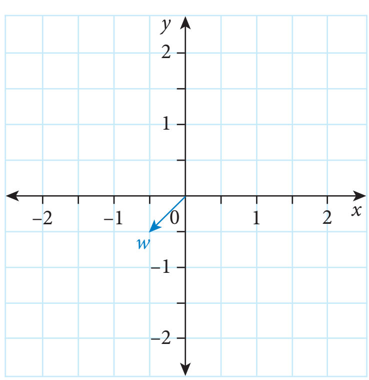


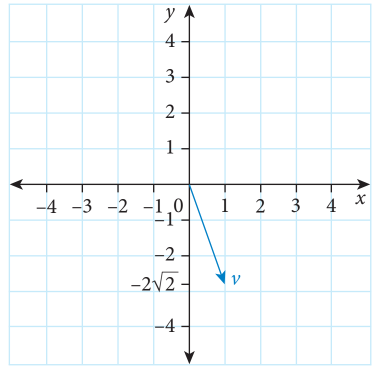




5 a $z = 3 + i$ b $w = -\frac{1}{2} - \frac{i}{2}$ c $v = 1 - 2i\sqrt{2}$



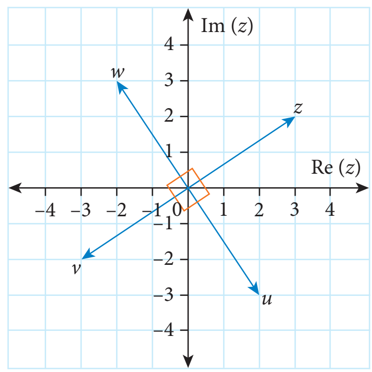




6 a i $w = -2 + 3i$ ii $v = -3 - 2i$

iii $u = 2 - 3i$

b



c Multiplying a vector by i rotates the vector 90° anticlockwise.

7 $z + \overline{z} = 2\text{Re}(z) \therefore \text{Re}(z) = 0$, i.e. z is purely imaginary.

8 $z + \overline{z} = 2\text{Re}(z)$ so $z + \overline{z}$ is real if $z + \overline{z} \neq 0$.

9 z must lie on the real axis.

10.06

1 a $\sqrt{34}$ b $\sqrt{5}$ c 2

d $\sqrt{3}$ e 1 f 1

2 a $\sqrt{13}$ b $\frac{1}{\sqrt{13}}$ c $\frac{1}{\sqrt{13}}$ d 13

3 a 2 b 3 c $\frac{1}{10}$ d 1

4 a $2\sqrt{17}$ b $\sqrt{221}$ c $2\sqrt{15}$

d $\frac{1}{2}$ e 1 f 3

5 a Proof b Proof

c Proof d Proof

6 a $\sqrt{x^2 + y^2}$ b $\frac{\sqrt{x^2 + y^2}}{x^2 + y^2}$ c 1

7 $|z| = |\overline{z}|$ because the modulus is the length of the vector. The vectors z and $\overline{z}$ are reflections so have the same length.

8 Show LHS = RHS.

9 Show LHS = RHS.

10 a Proof b Proof

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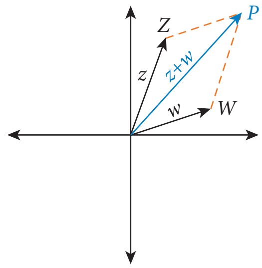
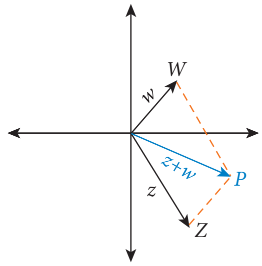
10.07

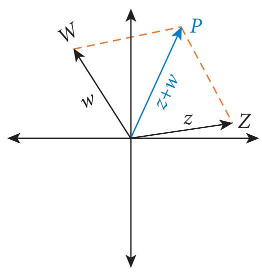
1 a



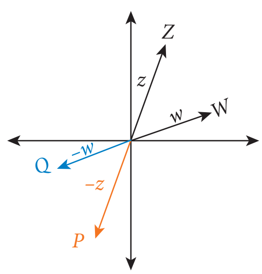
b



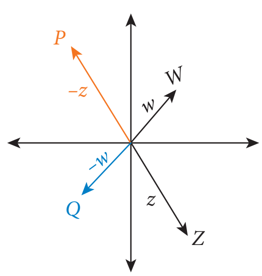
c



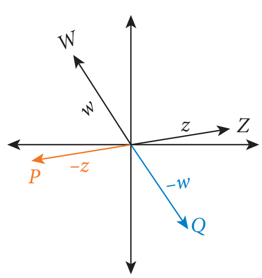
2 a



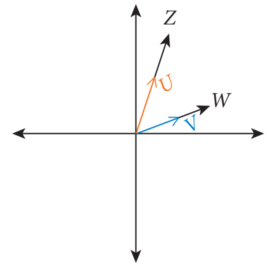
b



c



3 a



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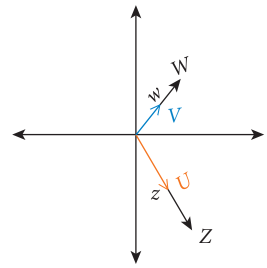
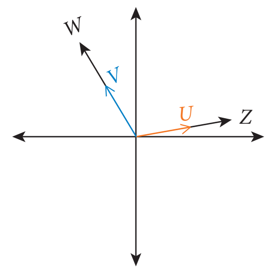
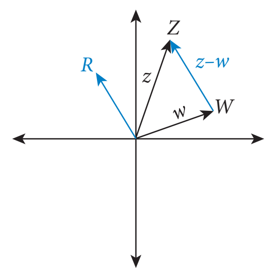


c



4 a



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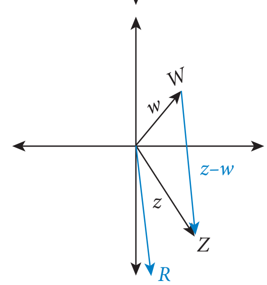

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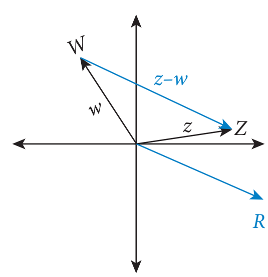

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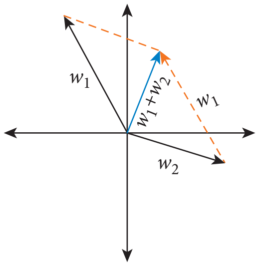
5 a $\mathbf{u}+\mathbf{v}$ b $\mathbf{v}-\mathbf{u}$ c $-\mathbf{u}-\mathbf{v}$

d $\mathbf{u}-\mathbf{v}$ e $\frac{1}{2}(\mathbf{u}+\mathbf{v})$

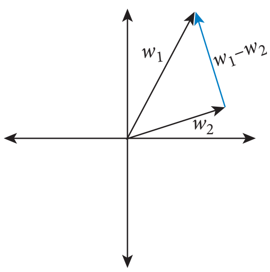
6 $1+3i$

7 a $\mathbf{z}_3=\mathbf{z}_1-\mathbf{z}_2$ b $\mathbf{z}_1=\mathbf{z}_2+\mathbf{z}_3$ c $\mathbf{z}_2=\mathbf{z}_1-\mathbf{z}_3$

8 a



b



The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

9 $\mathbf{z}_2=\mathbf{z}_1+\mathbf{w}$, etc.

10.08

1 Proofs

2 a $-z=-x-yi$ b $z^{-1}=\frac{x-yi}{x^2+y^2}$

c $0=0+0i$ d $1=1+0i$

3–7 Proofs

8 Yes, since $|z|^2=|z^2|$ if $z\neq 0$.

10.09

1 a $z=-1\pm i\sqrt{3}$ b $z=1\pm\sqrt{5}$

c $z=-2\pm 2i$ d $z=\frac{-1\pm i\sqrt{23}}{3}$

2 $x=-1\pm 3i$

3 $x=\frac{-7\pm i\sqrt{5}}{3}$

4 substitute, $\bar{\beta}=-5+i$

5 $m=-4$, other root $2+i$

6 a i $(x-1-2i)(x-1+2i)=0$

ii $x^2-2x+5=0$

b i $\left(x-\sqrt{3}-i\right)\left(x-\sqrt{3}+i\right)=0$

ii $x^2-2\sqrt{3}x+4=0$

c i $\left(x+4-i\sqrt{2}\right)\left(x+4+i\sqrt{2}\right)=0$

ii $x^2+8x+18=0$

d i $\left(x-\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}\right)\left(x-\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)=0$

ii $x^2-\sqrt{2}x+1=0$

e i $(x+5-6i)(x+5+6i)=0$

ii $x^2+10x+61=0$

f i $\left(x-\frac{\sqrt{3}}{2}-\frac{i}{2}\right)\left(x-\frac{\sqrt{3}}{2}+\frac{i}{2}\right)=0$

ii $x^2-\sqrt{3}x+1=0$

g i $\left(x+\frac{3}{5}-\frac{2i}{5}\right)\left(x+\frac{3}{5}+\frac{2i}{5}\right)=0$

ii $x^2+\frac{6}{5}x+\frac{13}{25}=0$

h i $\left(x+\frac{1}{\sqrt{3}}-\frac{i\sqrt{2}}{\sqrt{3}}\right)\left(x+\frac{1}{\sqrt{3}}+\frac{i\sqrt{2}}{\sqrt{3}}\right)=0$

ii $x^2+\frac{2}{\sqrt{3}}x+1=0$

7 a $k=1, n=-4, p=5$ b $k=1, n=-2, p=4$

c $k=2, n=1, p=2$ d $k=13, n=-10, p=13$

8 a $(z-1-i)(z-1+i)$

b $\left(z+1-i\sqrt{5}\right)\left(z+1+i\sqrt{5}\right)$

c $(z+2-i)(z+2+i)$

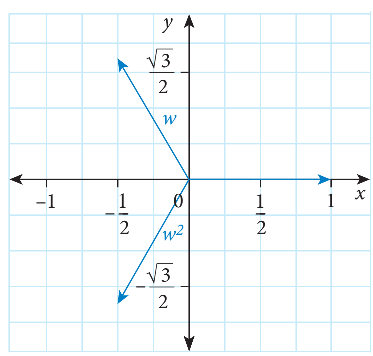
d $\left(z+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(z+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)$

9 a $z=1,-\frac{1}{2}\pm i\frac{\sqrt{3}}{2}$

10 a If $\omega=-\frac{1}{2}+\frac{i\sqrt{3}}{2}$ then $\omega^2=-\frac{1}{2}-\frac{i\sqrt{3}}{2}=\bar{\omega}$

b Proof. $|\omega|=\sqrt{\frac{1}{4}+\frac{3}{4}}$

11



$1, \omega, \omega^2$ are symmetrically placed around O.

CHAPTER 10 REVIEW

1 C

2 A

3 B

4 D

5 C

6 a $9i$ b $3i\sqrt{2}$ c $4i\sqrt{3}$ d $6i\sqrt{6}$

7 a $z=\pm 6i$ b $z=\pm 3i$ c $z=\pm\frac{i}{2}$

8 a $-i$ b 1 c i

d -1 e $-i$

9 a $\text{Re}(z) = 5, \text{Im}(z) = -\sqrt{5}$

b $\text{Re}(z) = \frac{1}{2}, \text{Im}(z) = \frac{\sqrt{3}}{2}$

c $\text{Re}(z) = x - 2y, \text{Im}(z) = 3y + x$

10 a $x = -2 \pm i\sqrt{2}$ b $x = \frac{5 \pm i\sqrt{11}}{3}$ c $x = \frac{-1 \pm i\sqrt{7}}{2}$

11 a $-4 - 7i$ b $2 - 5i$ c $2x + y + yi - 3xi$

12 a $(x + 2 - i)(x + 2 + i)$

b $(z - 5 - 2i)(z - 5 + 2i)$

c $\left(w + \frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\left(w + \frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

13 a $x^2 - 6x + 10 = 0$

b $x^2 - 2x + 7 = 0$

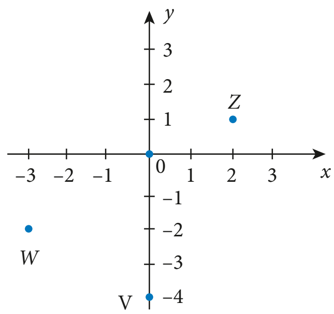
c $4x^2 - 8x + 7 = 0$

14 $x = 2.4, y = -0.8$

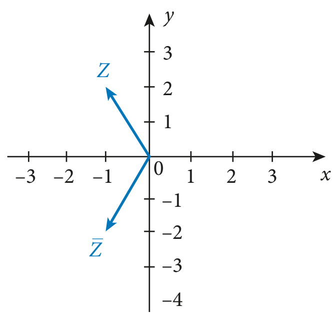
15 a $-2 + 2i$ b $31 + 5i$ c $5 - 12i$

16 a $\frac{1+3i}{5}$ b $\frac{\sqrt{2}-i}{3}$ c $-2i$

17



18



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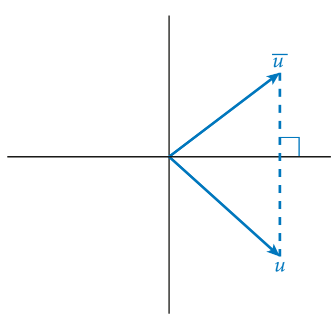
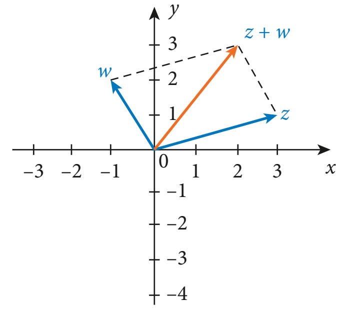


20 a $\sqrt{13}$ b $\frac{1}{2}$ c $\frac{1}{\sqrt{7}}$

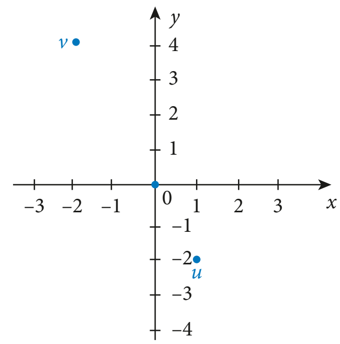
21 $\sqrt{5x^2 + 2xy + 2y^2}$

22

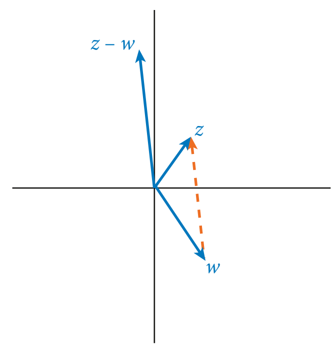


23 a $U(1, -2), V(-2, 4)$

b



24



25 a $\frac{1+i\sqrt{2}}{3}$ b $-\frac{1}{2}i$ c $3 - 4i$

26 a Proof b Proof

27 a $x = \pm 2i$, imaginary b $z = \frac{1}{3}, 3$ real

c $w = 1 \pm i$ complex

28 $4 + i$

29 $\left(z + 3 - i\sqrt{2}\right)\left(z + 3 + i\sqrt{2}\right) = 0$

30 a $44 + 23i$ b 10

c $\frac{1 + 2i\sqrt{2}}{3}$ d $3 + 4i$

31 a $\sqrt{3} - 1 - i\left(1 + \sqrt{3}\right)$ b $\sqrt{3} - 1$

c $-1 - \sqrt{3}$ d $2\sqrt{2}$

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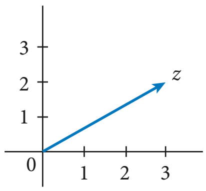
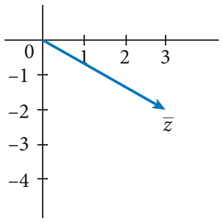
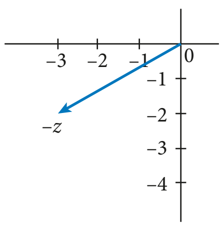
32 a



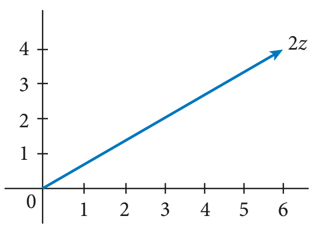
b



c



d



33 a $z = \pm 5i$ **b** $z = 5 \pm 3i$

c $w = 0$ or $w = -3$ **d** $u = -2 \pm \dfrac{\sqrt{2}}{2}i$

e $z = \dfrac{-7 \pm i\sqrt{11}}{6}$ **f** $w = \pm 3, \pm 2i$

34 $a(z^2 + 4z + 8)$

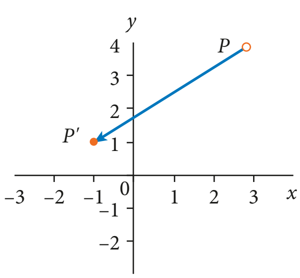
35 $x = \dfrac{3 \pm i\sqrt{11}}{2}$

36 $z = 2, -1 \pm i\sqrt{3}$

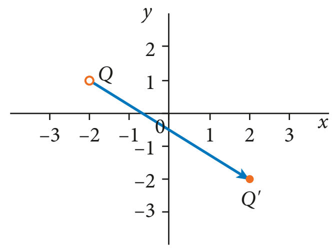
37 $X(2 + 4i), M\left(\dfrac{3}{2} + \dfrac{5}{2}i\right)$

11.01

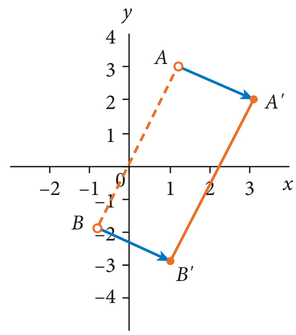
1 a



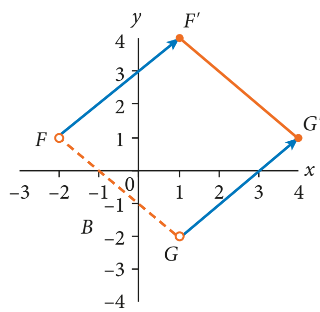
b



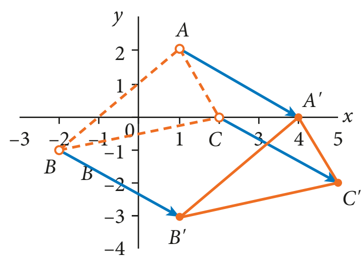
c



d



e



2 a $A'(-5, -2)$ **b** $B'(-2, 6)$ **c** $C'(-4, 3)$
d $D'(-3, 0)$ **e** $E'(-7, 9)$

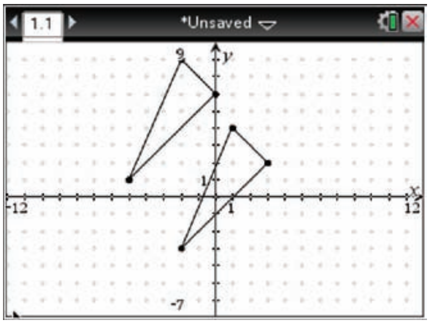
3 a $A'(2, 2)$ **b** $B'(-6, -6)$ **c** $C'(3, -5)$
d $D'(5, -5)$ **e** $E'(-5, 6)$

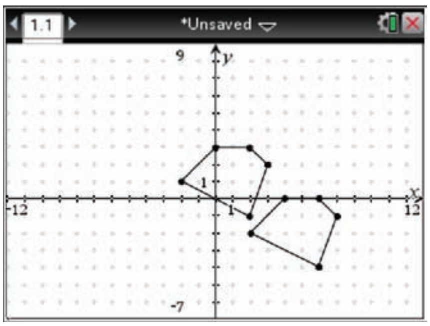
4 a $A'(1, 1)$ **b** $B'(8, -9)$ **c** $C'(-3, -4)$
d $D'(-2, -11)$ **e** $E'(3, -10)$

5 $\begin{bmatrix} -4 \\ -1 \end{bmatrix}$

6 $\begin{bmatrix} 7 \\ -4 \end{bmatrix}$, 7 right and 4 down

7 $\begin{bmatrix} 4 \\ -6 \end{bmatrix}$, 4 across and 6 down

8 

9 

10 a $P'(2, 4)$ b $\begin{bmatrix} 1 \\ 6 \end{bmatrix}$ c $\begin{bmatrix} a+c \\ b+d \end{bmatrix}$

11 a $\mathbf{I} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ b $\mathbf{T}_i = \begin{bmatrix} -a \\ -b \end{bmatrix}$

12 For $\mathbf{S} = \begin{bmatrix} a \\ b \end{bmatrix}$ and $\mathbf{T} = \begin{bmatrix} c \\ d \end{bmatrix}$, $\mathrm{S} \circ \mathrm{T} = \begin{bmatrix} a+c \\ b+d \end{bmatrix} = \begin{bmatrix} c+a \\ d+b \end{bmatrix}$

$= \mathrm{T} \circ \mathrm{S}$ QED

11.02

1 a Proof b $A'(6, -5)$, $B'(2, -5)$, $C'(2, 3)$

c A right-angled triangle

2 a Proof

b $A'(17, -13)$, $B'(6, 0)$, $C'(-10, -11)$, $D'(1, -2)$

c A parallelogram

3 a Proof

b $A'(15, -20)$, $B'(10, -10)$, $C'(30, 0)$, $D'(35, -10)$

c A parallelogram

4 a $\begin{bmatrix} 2 & -4 \\ 4 & 3 \end{bmatrix}$ b $A'(-14, 5)$, $B'(-16, 23)$, $C'(18, 3)$

5 a $\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$ b $A'(3, -3)$, $B'(8, -8)$, $C'(1, -1)$

6 a $\begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$ b $A'(-7, 1)$, $B'(8, 10)$, $C'(-1, 1)$

7 a $\begin{bmatrix} -2 & 3 \\ 4 & 3 \end{bmatrix}$

b $A'(-5, -17)$, $B'(-1, -7)$, $C'(12, -6)$, $D'(8, -16)$

8 a $\begin{bmatrix} 5 & -2 \\ 3 & 4 \end{bmatrix}$

b $A'(-5, 23)$, $B'(-16, 6)$, $C'(33, -1)$, $D'(23, 45)$

9 a $A'(2, 5)$, $B'(4, -5)$, $C'(5, -19)$, $D'(3, -9)$ and $A'(3, 8)$, $B'(11, -4)$, $C'(18, -22)$, $D'(10, -10)$

b Both parallelograms.

10 a $A'(2a + 3b, 2c + 3d)$, $B'(3a + 6b, 3c + 6d)$, $C'(5a + 4b, 5c + 4d)$, $D'(4a + b, 4c + d)$

b Proof

11.03

1 a $(x, y) \to (4x, 3y)$ b $(x, y) \to (1.2x, 1.2y)$

c $(x, y) \to (3x, 0.5y)$ d $(x, y) \to (0.25x, 3y)$

2 a $\begin{bmatrix} 1.5 & 0 \\ 0 & 2 \end{bmatrix}$

b $A'(4.5, 8)$, $B'(6, 16)$, $C'(10.5, 12)$

c A triangle

3 a $\begin{bmatrix} 0.4 & 0 \\ 0 & 0.8 \end{bmatrix}$

b $A'(-4, -6.4)$, $B'(-2, 3.2)$, $C'(7.6, -4.8)$, $D'(5.6, -14.4)$

c A parallelogram

4 a $\begin{bmatrix} 0.7 & 0 \\ 0 & 0.9 \end{bmatrix}$

b $A'(-7, -7.2)$, $B'(-2.8, 1.8)$, $C'(4.2, -3.6)$, $D'(0, -12.6)$

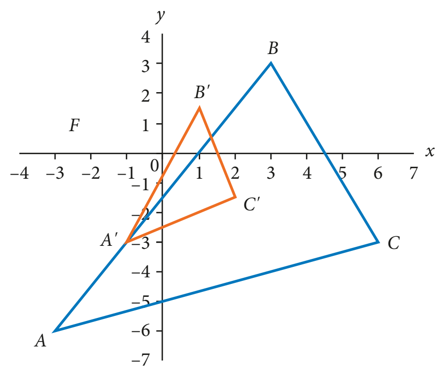
c A parallelogram

5 a $\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$

b $A'(-6, 2)$, $B'(0, 8)$, $C'(9, 8)$, $D'(9, -2)$, $E'(3, -4)$

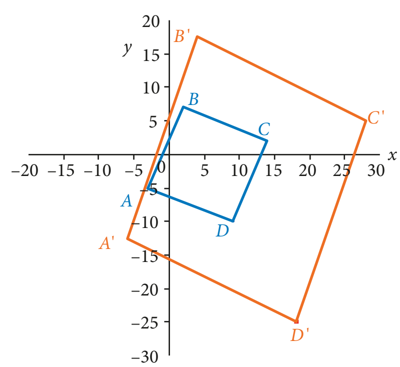
c A pentagon

6 a $A'(-1, -3)$, $B'(1, 1.5)$, $C'(2, -1.5)$

b 

c They are triangles with different angles.

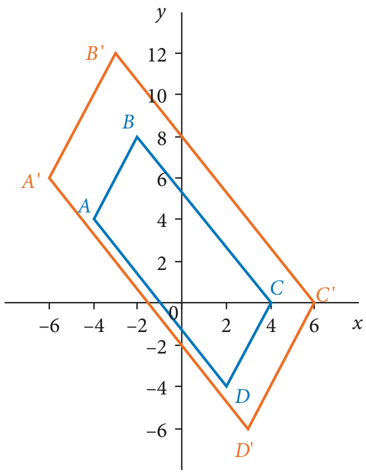
7 a $A'(-6, -12.5)$, $B'(4, 17.5)$, $C'(28, 5)$, $D'(18, -25)$

b 

c The square has changed to a parallelogram.

8 a $A'(-6, 6)$, $B'(-3, 12)$, $C'(6, 0)$, $D'(3, -6)$

b



c They are similar.

11.04

1 $P'(3-3\sqrt{3}, -(3+3\sqrt{3}))$

2 $P'(0, -4)$

3 $A'(-0.83, -2.70)$, $B'(-1.60, -7.65)$, $C'(-5.77, -1.93)$, $D'(0.83, 2.7)$

4 $A'(0.64, -7.59)$, $B'(-3.25, -13.05)$, $C'(-5.71, -4.17)$, $D'(-1.46, 4.88)$

5 a $\begin{bmatrix} 0.80 & 0.60 \\ -0.60 & 0.80 \end{bmatrix}$

b $A'(6.2, 1.59)$, $B'(9.61, 2.78)$, $C'(4.99, -5.01)$

6 a $\begin{bmatrix} -0.73 & -0.68 \\ 0.68 & -0.73 \end{bmatrix}$

b $A'(1.56, -4.19)$, $B'(-2.63, -5.75)$, $C'(-8.38, -3.12)$, $D'(-7.75, -0.98)$, $E'(-5.55, -3.02)$

7 a $\begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$

b $A'(2\sqrt{2}, 0)$, $B'(-2\sqrt{2}, -\sqrt{2})$, $C'(-6\sqrt{2}, 3\sqrt{2})$, $D'(-2\sqrt{2}, 4\sqrt{2})$

8 a $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ b $-60°$ or $300°$

c $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

9 a $\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$ b $-\frac{7\pi}{4}$ or $\frac{\pi}{4}$

c $\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

10 a $-\frac{\pi}{6}, \frac{11\pi}{6}$, $-30°$ or $330°$ b $\mathbf{R}_{-\theta} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

11.05

1 a $P'(-11, 2)$ b $P'(7, 3)$

c $P'(9, -3)$ d $P'(15, 20)$

2 $A'(2, -1)$, $B'(3, 1)$, $C'(-0.4, -2.8)$

3 $A'(1, -7)$, $B'(-4, 3)$, $C'(-4.4, -4.2)$

4 $A'(12.4, 6.8)$, $B'(3.4, -6.2)$, $C'(-9, -13)$, $D'(-11, -2)$

5 $A'(-6.2, 1.6)$, $B'(-11, 3)$, $C'(-12.4, -1.8)$, $D'(-6.2, -3.4)$, $E'(-4, -3)$

6 $A'(6.33, 0.96)$, $B'(10.79, 0.70)$, $C'(7.56, -0.90)$

7 $A'(6.36, 0.71)$, $B'(7.07, 14.14)$, $C'(14.14, 14.14)$

8 $A'(-4.23, -3.33)$, $B'(-6.56, 2.63)$, $C'(-16.49, 1.44)$, $D'(-15.89, -3.53)$

9 $A'(-1.12, -4.97)$, $B'(1.82, -10.66)$, $C'(11.57, -8.43)$, $D'(10.46, -3.56)$, $E'(6.59, -7.31)$

10 Proof

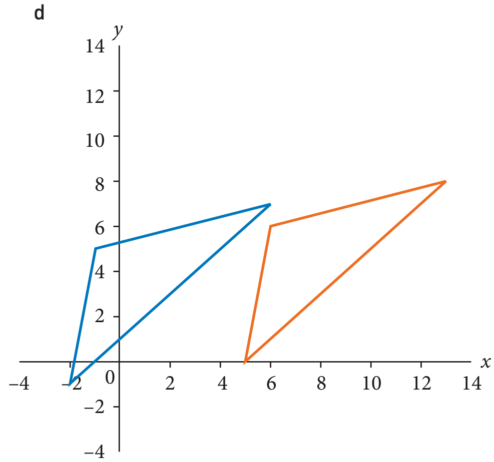
11.06

1 a $\mathbf{T}_1 \circ \mathbf{T}_2: (x, y) \rightarrow (1 + x, -3 + y)$

b $\mathbf{T}_3 \circ \mathbf{T}_2 = \begin{bmatrix} 2 \\ -10 \end{bmatrix}$

c $A'(5, 0)$, $B'(6, 6)$, $C'(13, 8)$

d

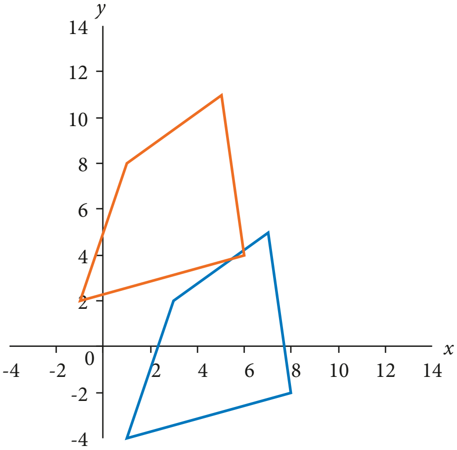


2 a $\mathbf{T}_1 \circ \mathbf{T}_2: (x, y) \to (1 + x, -2 + y)$

b $\mathbf{T}_3 \circ \mathbf{T}_2 = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$

c $A'(-1, 2), B'(1, 8), C'(5, 11), D'(6, 4)$

d



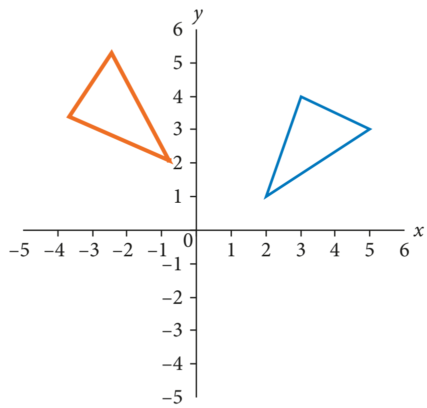
3 a $(x, y) \to (0.1045x - 0.9945y, 0.9945x + 0.1045y)$

b $(x, y) \to (0.1045x - 0.9945y, 0.9945x + 0.1045y)$

c Yes

d $A'(-3.66, 3.4), B'(-2.46, 5.29), C'(-0.79, 2.09)$

e

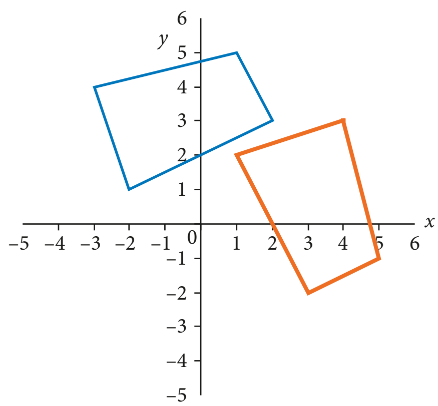


4 a $(x, y) \to (y, -x)$ b $(x, y) \to (-y, x)$

c No

d $A'(5, -1), B'(3, -2), C'(1, 2), D'(4, 3)$

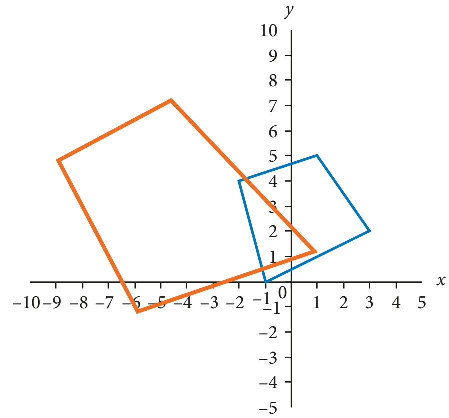
e



5 a $\begin{bmatrix} -0.9 & -1.2 \\ -1.6 & 1.2 \end{bmatrix}$ b $\begin{bmatrix} -0.9 & -1.6 \\ -1.2 & 1.2 \end{bmatrix}$ c No

d $A'(-4.6, 7.2), B'(-8.9, 4.8), C'(-5.9, -1.2), D'(0.9, 1.2)$

e

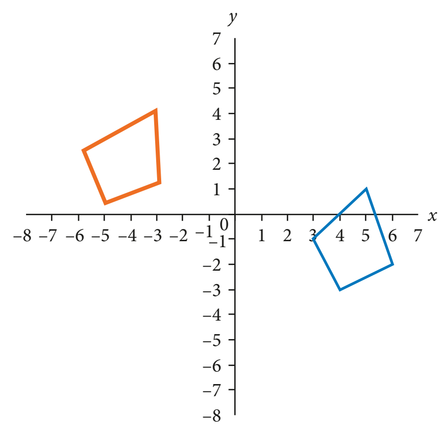


6 a $\begin{bmatrix} -0.7431 & 0.6691 \\ 0.6691 & 0.7431 \end{bmatrix}$ b $\begin{bmatrix} 0.9272 & -0.3746 \\ -0.3746 & -0.9272 \end{bmatrix}$

c No

d $A'(-2.9, 1.26), B'(-3.05, 4.09), C'(-5.8, 2.53), D'(-4.98, 0.45)$

e



7 a $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ b $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ c Yes

8–10 Proofs

11.07

1 a $\mathbf{T} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \mathbf{T}^{-1} = \begin{bmatrix} -3 \\ -5 \end{bmatrix}, \mathbf{T} \circ \mathbf{T}^{-1} = \mathbf{T}^{-1} \circ \mathbf{T} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

b $\mathbf{T} = \begin{bmatrix} -5 \\ -4 \end{bmatrix}, \mathbf{T}^{-1} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}, \mathbf{T} \circ \mathbf{T}^{-1} = \mathbf{T}^{-1} \circ \mathbf{T} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

c $\mathbf{T} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}, \mathbf{T}^{-1} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}, \mathbf{T} \circ \mathbf{T}^{-1} = \mathbf{T}^{-1} \circ \mathbf{T} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

d $\mathbf{T} = \begin{bmatrix} -7 \\ 5 \end{bmatrix}, \mathbf{T}^{-1} = \begin{bmatrix} 7 \\ -5 \end{bmatrix}, \mathbf{T} \circ \mathbf{T}^{-1} = \mathbf{T}^{-1} \circ \mathbf{T} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

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2 $\mathbf{T}=\begin{bmatrix}\frac{1}{3} & 0\\ 0 & \frac{1}{2}\end{bmatrix}$, $\mathbf{T}^{-1}=\begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix}$, $\mathbf{T}\circ\mathbf{T}^{-1}=\mathbf{T}^{-1}\circ\mathbf{T}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

3 $\mathbf{T}=\begin{bmatrix}2\frac{1}{2} & 0\\ 0 & 2\frac{1}{2}\end{bmatrix}$, $\mathbf{T}^{-1}=\begin{bmatrix}\frac{2}{5} & 0\\ 0 & \frac{2}{5}\end{bmatrix}$,

$\mathbf{T}\circ\mathbf{T}^{-1}=\mathbf{T}^{-1}\circ\mathbf{T}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

4 $\mathbf{T}=\begin{bmatrix}-\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$, $\mathbf{T}^{-1}=\begin{bmatrix}-\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$,

$\mathbf{T}\circ\mathbf{T}^{-1}=\mathbf{T}^{-1}\circ\mathbf{T}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

5 $\mathbf{T}=\begin{bmatrix}-\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$, $\mathbf{T}^{-1}=\begin{bmatrix}-\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$,

$\mathbf{T}\circ\mathbf{T}^{-1}=\mathbf{T}^{-1}\circ\mathbf{T}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

6 $\mathbf{T}=\begin{bmatrix}-\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{bmatrix}$, $\mathbf{T}^{-1}=\begin{bmatrix}-\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{bmatrix}$,

$\mathbf{T}\circ\mathbf{T}^{-1}=\mathbf{T}^{-1}\circ\mathbf{T}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

7 $\mathbf{T}=\begin{bmatrix}\frac{4}{5} & -\frac{3}{5}\\ -\frac{3}{5} & -\frac{4}{5}\end{bmatrix}$, $\mathbf{T}^{-1}=\begin{bmatrix}\frac{4}{5} & -\frac{3}{5}\\ -\frac{3}{5} & -\frac{4}{5}\end{bmatrix}$,

$\mathbf{T}\circ\mathbf{T}^{-1}=\mathbf{T}^{-1}\circ\mathbf{T}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

8 a $\mathbf{T}_1\circ\mathbf{T}_2=\begin{bmatrix}\frac{\sqrt{6}-\sqrt{2}}{4} & \frac{-\sqrt{6}-\sqrt{2}}{4}\\ \frac{\sqrt{6}+\sqrt{2}}{4} & \frac{\sqrt{6}-\sqrt{2}}{4}\end{bmatrix}$,

$(\mathbf{T}_1\circ\mathbf{T}_2)^{-1}=\begin{bmatrix}\frac{\sqrt{6}-\sqrt{2}}{4} & \frac{\sqrt{6}+\sqrt{2}}{4}\\ \frac{-\sqrt{6}-\sqrt{2}}{4} & \frac{\sqrt{6}-\sqrt{2}}{4}\end{bmatrix}$

$=\mathbf{T}_2^{-1}\circ\mathbf{T}_1^{-1}=\mathbf{T}_1^{-1}\circ\mathbf{T}_2^{-1}$

The composition of the inverses is the same in either order.

b $\mathbf{T}_1\circ\mathbf{T}_2=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$,

$(\mathbf{T}_1\circ\mathbf{T}_2)^{-1}=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}=\mathbf{T}_2^{-1}\circ\mathbf{T}_1^{-1}$,

$\mathbf{T}_1^{-1}\circ\mathbf{T}_2^{-1}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$.

The composition of the inverses is different when the order is reversed.

9 a Both **I** b Both **I** c $\mathbf{T}_2$
d $\mathbf{T}_2^{-1}$ e Proof

10 Proof

11.08

1 a 182 b 360 c 300

2 A reflection in the line $y = -x$.

3 A rotation of –45°.

4 A dilation with factors 8 and 6 in the x and y directions.

5 A reflection in the line $y = -x$.

6 a 4 b $\begin{bmatrix}1 & -2\\ 2 & 3\end{bmatrix}$

c $A'(3, -8)$, $B'(-2, 10)$, $C'(0, 14)$

d 28 e 7

f The area of the image is det **T** (= 7 in this case) times the area of the object.

7 a 26 b $\begin{bmatrix}-2 & 1\\ 2 & -2\end{bmatrix}$ c 52

8 a 140 b 100 c 1080

9 a $\begin{bmatrix}\cos(50°) & -\sin(50°)\\ \sin(50°) & \cos(50°)\end{bmatrix}$

b A rotation through an angle of 50°.

10 a $\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$ b A rotation through an angle of 90°.

11–12 Proofs

CHAPTER 11 REVIEW

1 C

2 E

3 B

4 A

5 B

6 A

7 $A'(4, 2)$, $B'(8, 4)$, $C'(11, 0)$, $D'(15, -5)$

8 a Proof

b $A'(-3, 4)$, $B'(18, 18)$, $C'(3, 0)$, $D'(-18, -14)$

c A parallelogram

9 a $(x, y) \to (3x, 1.5y)$ b $\begin{bmatrix}3 & 0\\ 0 & 1.5\end{bmatrix}$

c $A'(-9, -3)$, $B'(9, 9)$, $C'(-3, 13.5)$

d A non-right-angled scalene triangle

10 a $\begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$

b $A\left(\frac{2+\sqrt{3}}{2}, \frac{2\sqrt{3}-1}{2}\right)$, $B\left(1-2\sqrt{3}, 2+\sqrt{3}\right)$,

$C\left(\frac{-(5+4\sqrt{3})}{2}, \frac{4-5\sqrt{3}}{2}\right)$, $D\left(\frac{\sqrt{3}-5}{2}, \frac{-(1+5\sqrt{3})}{2}\right)$

c They are congruent.

11 $A'(-4.95, -0.71)$, $B'(-6.36, -4.95)$, $C'(-3.54, -6.36)$, $D'(0.71, -4.95)$

12 $A'(5, 6)$, $B'(14, 4)$, $C'(12, 3)$

13 a $A'(1.6, 6.6)$, $B'(-4.4, -6.9)$, $C'(2.4, -5.1)$ **b** No

14 a A translation 3 right and 4 down

b A dilation with x factor 2 and y factor $\frac{2}{3}$

c A rotation of $-\frac{4\pi}{3}$.

d A reflection in the line through the origin with inclination 25°.

15 225

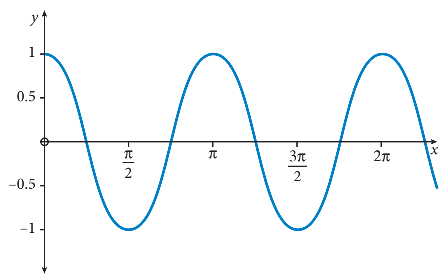
16 Proof

17 a 5.5 **b** Proof

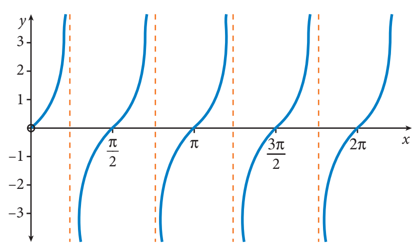
18 Proof

12.01

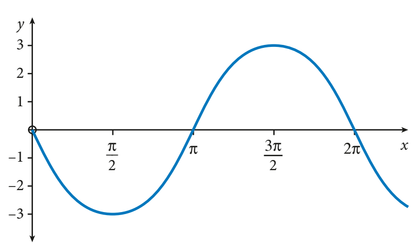
1 a



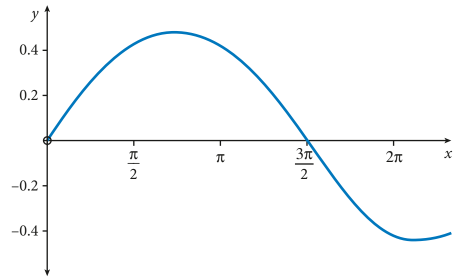
b



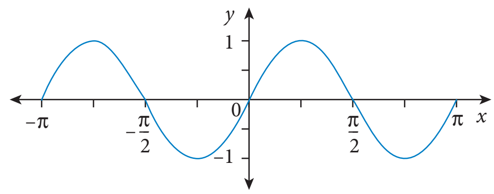
c



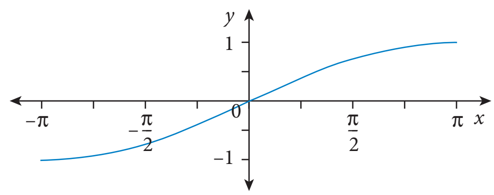
d



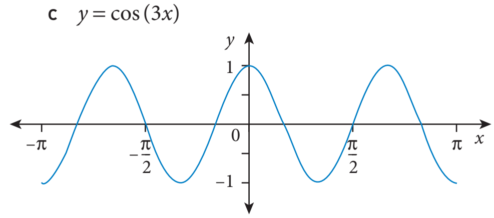
2 a $y = \sin(2x)$



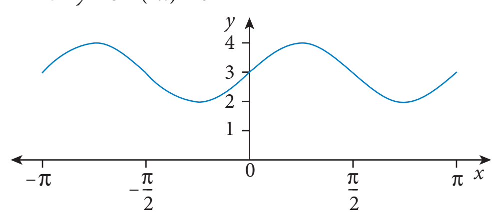
b $y = \sin\left(\frac{1}{2}x\right)$



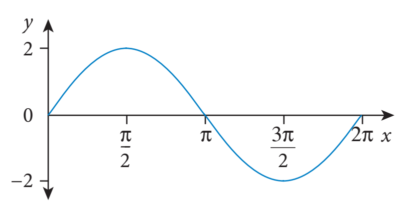
c $y = \cos(3x)$



d $y = \sin(2x) + 3$

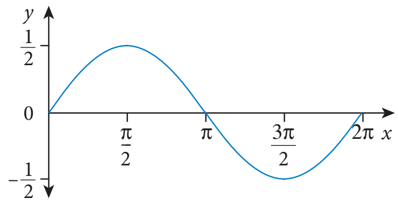


3 a $y = 2\sin(x)$

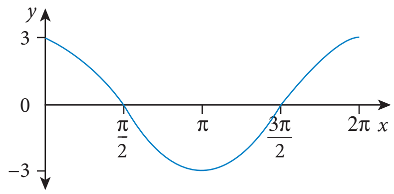


9780170250276

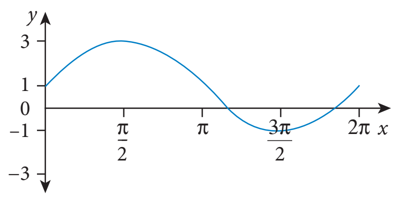
b $y=\frac{1}{2}\sin(x)$



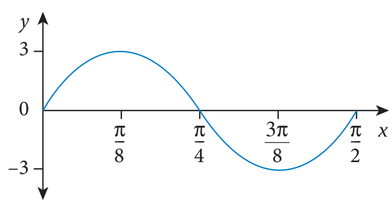
c $y=3\cos(x)$



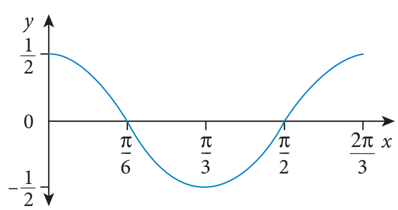
d $y=2\sin(x)+1$



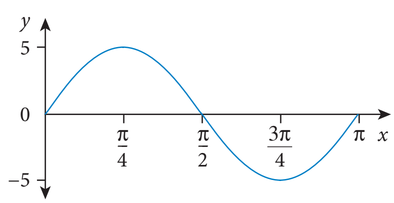
4 **a** $y=3\sin(4x)$



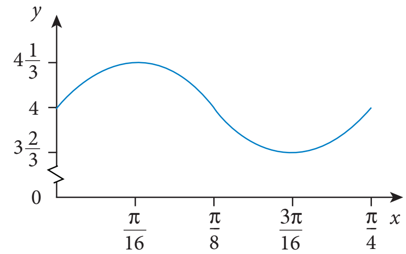
b $y=\frac{1}{2}\cos(3x)$



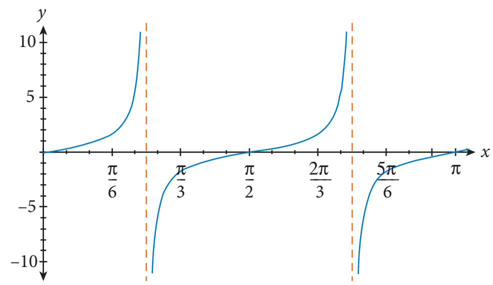
c $y=5\sin(2x)$



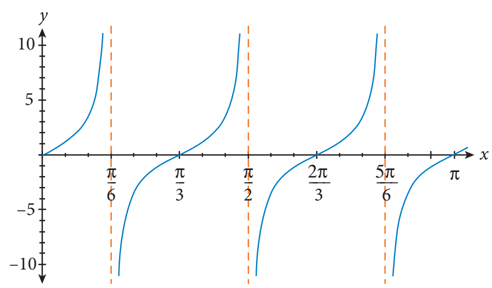
d $y=\frac{1}{3}\sin(8x)+4$



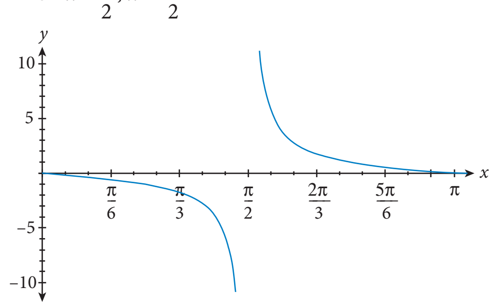
5 **a** $x=\frac{\pi}{4}, x=\frac{3\pi}{4}$



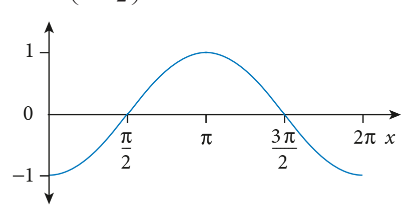
b $x=\frac{\pi}{6}, x=\frac{\pi}{2}$



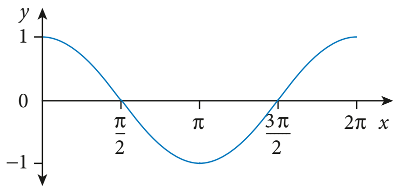
c $x=\frac{\pi}{2}, x=\frac{3\pi}{2}$



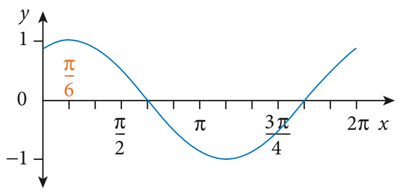
6 **a** $y=\sin\left(x-\frac{\pi}{2}\right)$



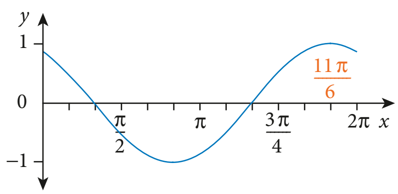
b $y = \sin\left(x + \frac{\pi}{2}\right)$



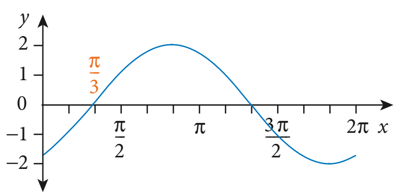
c $y = \cos\left(x - \frac{\pi}{6}\right)$



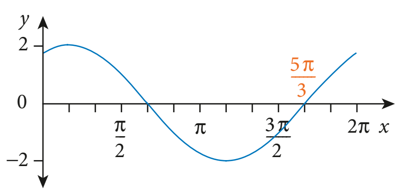
d $y = \cos\left(x + \frac{\pi}{6}\right)$



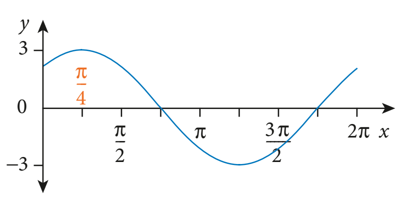
e $y = 2\sin\left(x - \frac{\pi}{3}\right)$



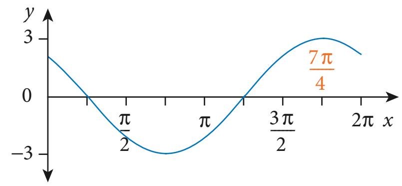
f $y = 2\sin\left(x + \frac{\pi}{3}\right)$



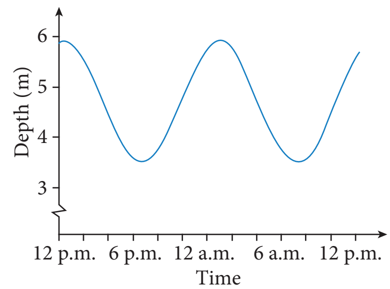
g $y = 3\cos\left(x - \frac{\pi}{4}\right)$



h $y = 3\cos\left(x + \frac{\pi}{4}\right)$



7 a $d = 4.7 + 1.2\cos(0.5t - 0.21)$



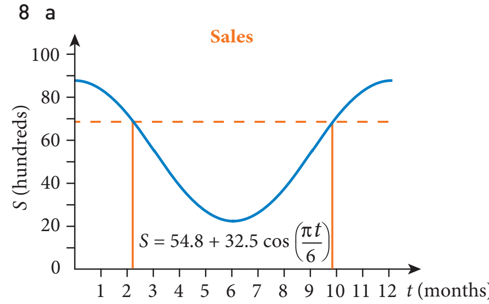
b About 6:42 p.m.

c About 5 h 16

d About 12 h 34 m

e About 25 minutes

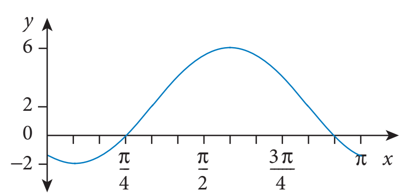
8 a



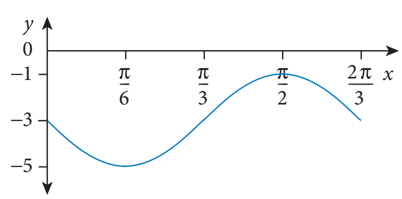
b Jan, Feb, Oct, Nov and Dec

12.02

1 a $y = 4\sin\left[2\left(x - \frac{\pi}{3}\right)\right] + 2$



b $y = 2\cos\left[3\left(x + \frac{\pi}{6}\right)\right] - 3$



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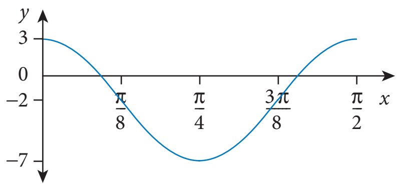
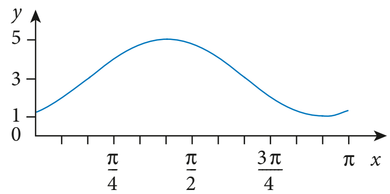
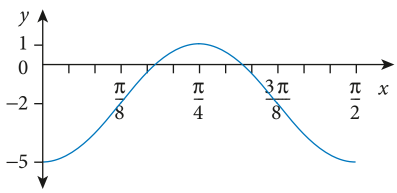
c $y = 5\sin\left[4\left(x + \frac{\pi}{8}\right)\right] - 2$



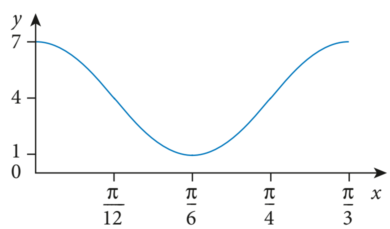
2 a $y = 2\sin\left(2x - \frac{\pi}{3}\right) + 3$



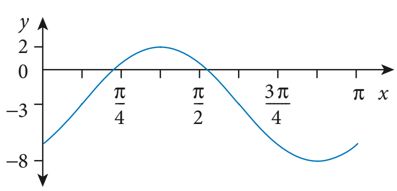
b $y = 3\cos(4x + \pi) - 2$



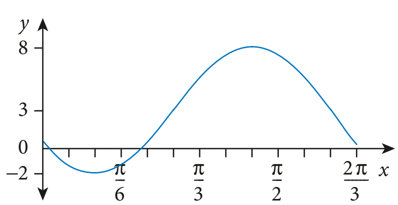
c $y = 4 - 3\sin\left(6x + \frac{3\pi}{2}\right)$



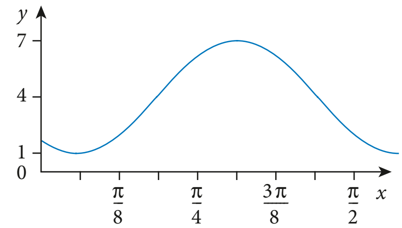
d $y = 5\cos\left(2x - \frac{3\pi}{4}\right) - 3$



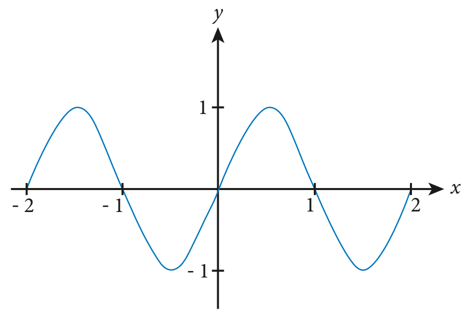
e $y = 3 - 5\cos\left(3x - \frac{\pi}{3}\right)$



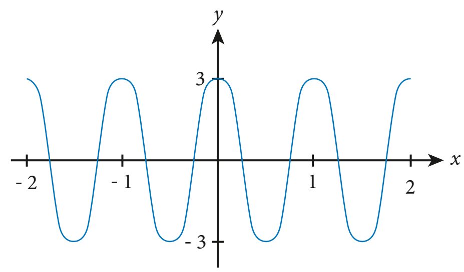
f $y = 4 - 3\sin\left(4x + \frac{\pi}{4}\right)$



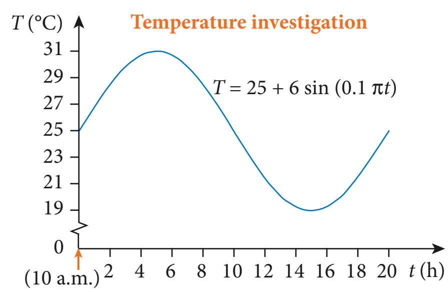
3 a



b



4 a



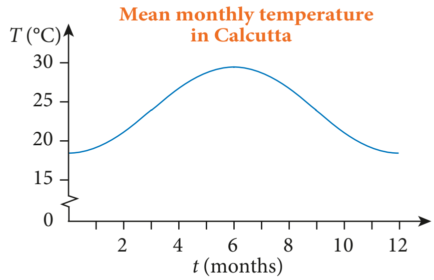
b 31°C, 19°C **c** 3 p.m., 1 a.m.

d **i** 11:05 a.m., 6:55 p.m., …

ii 11:08 p.m., 2:52 a.m., …

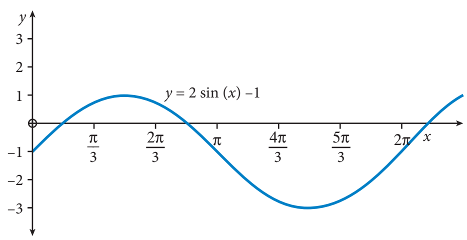
e Because the period was not 24 hours and it is unusual to have the maximum so early in the day.

5 $T = 23.875 + 5.575 \cos\left(\frac{\pi t}{6}\right)$.



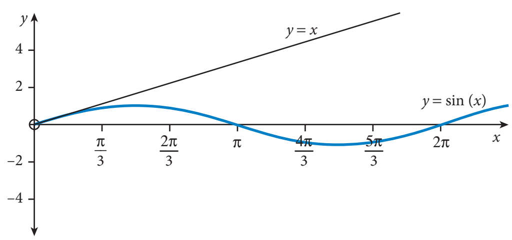
12.03

1

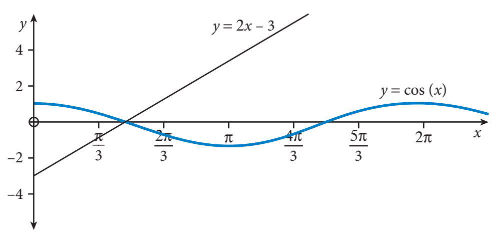


Solutions: $x = 0.52$, $x = 2.62$

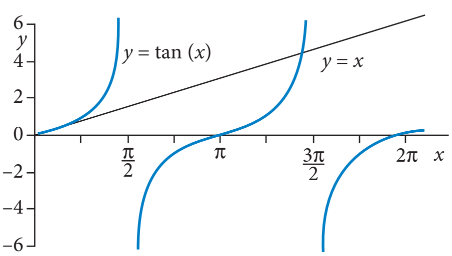
2 $x = 0$



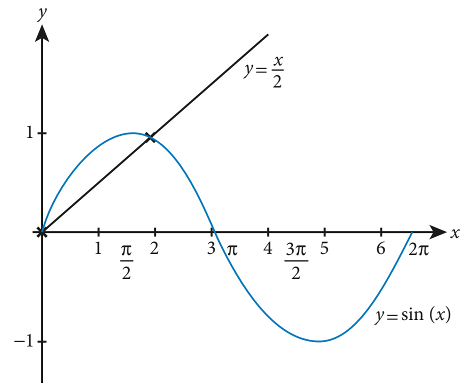
3 $x = 1.5$



4 $x = 0, 4.5$

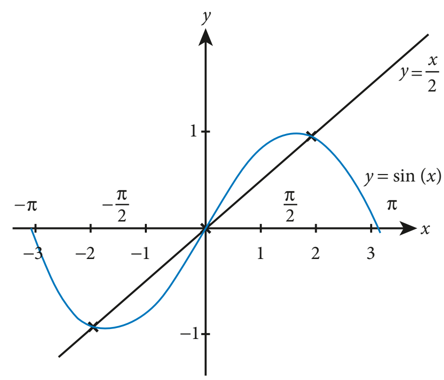


5 a



There are 2 points of intersection, so there are 2 solutions to the equation.

b



There are 3 points of intersection, so there are 3 solutions to the equation.

6 About 1.4 m

12.04

From **4** onwards, $n \in \mathbf{Z}$.

1 a $\frac{\pi}{6}, \frac{5\pi}{6}$ b $\frac{7\pi}{24}, \frac{19\pi}{24}, \frac{31\pi}{24}, \frac{43\pi}{24}$

c $\frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$

2 a $x = -\pi, \pi$ b $x = \frac{\pi}{2}$ c $x = -\frac{7\pi}{4}, -\frac{3\pi}{4}$

d $x = 0, 2\pi$ e $x = -\pi, \pi$ f $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

g $x = -\frac{\pi}{4}, \frac{3\pi}{4}$ h $x = \frac{\pi}{6}, \frac{5\pi}{6}$ i $x = -\frac{\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$

j $x = \frac{11\pi}{6}, \frac{13\pi}{6}$

3 a $\theta = 45°, 225°$ b $\theta = 30°, 150°$

c $\theta = 150°, 210°$ d $\theta = 225°, 315°$

e $\theta = 90°, 270°$ f $\theta = 60°, 240°$

g $\theta = 120°, 240°$ h $\theta = 150°, 330°$

i $\theta = 50°, 130°$ j $\theta = 115°, 245°$

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4 a $x = 2n\pi - \frac{\pi}{3}, 2n\pi + \frac{4\pi}{3}$

b $x = 2n\pi \pm \frac{\pi}{6}$

c $x = n\pi + \frac{\pi}{6}$

5 a $x = 0.3398, 2.8018$ b $x = 1.8235$

c $x = 0.3805$ d $x = 2.1518$

e $x = 0.4429, 2.6987$ f $x = 1.7197$

6 a $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ b $x = \frac{\pi}{2}, \frac{3\pi}{2}$

c $x = \frac{\pi}{4}, \frac{5\pi}{4}$ d $x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

e $x = \frac{\pi}{4}, \frac{5\pi}{4}$ f $x = \frac{\pi}{3}, \frac{4\pi}{3}$

7 a $x = -\frac{\pi}{6} + n\pi$ or $\frac{2\pi}{3} + n\pi$

b $x = \frac{\pi}{18} + \frac{2n\pi}{3}$ or $\frac{5\pi}{18} + \frac{2n\pi}{3}$

c $x = \frac{5\pi}{24} + n\pi$ or $\frac{17\pi}{24} + n\pi$

d $x = \frac{7\pi}{6} + n\pi$ or $\frac{4\pi}{3} + n\pi$

e $x = n\pi$ or $\frac{3\pi}{4} + n\pi$

8 a −2.46, −0.16, 0.68, 2.98

b −2.40, −1.35, −0.30, 0.75, 1.79, 2.84

c −1.95, −1.59, −0.38, −0.02, 1.19, 1.56, 2.76, 3.13

d −3.12, −1.45, −1.02, 0.64, 1.07, 2.74

e −2.46, −2.11, −1.20, −0.86, 0.06, 0.40, 1.31, 1.66, 2.57, 2.91

9 $x \approx 0.47, -1.51$

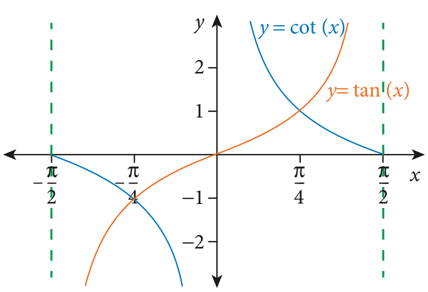
10 a 0.34 m

b 0.37 m

c 0.38 m when $t = \frac{\pi}{8}$

12.05

1



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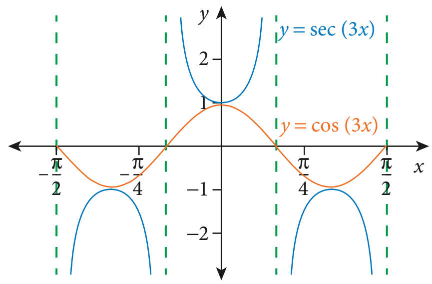

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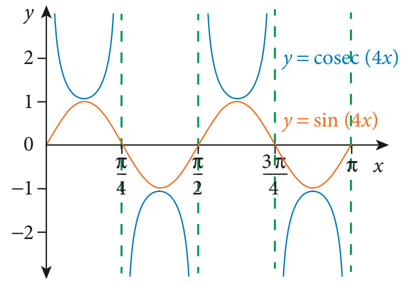

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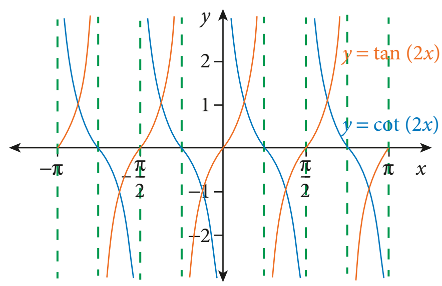

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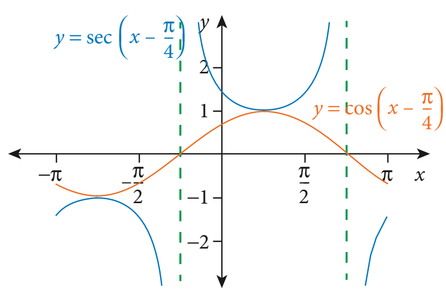

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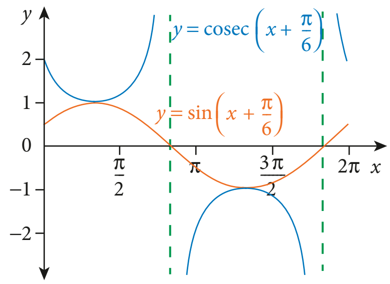

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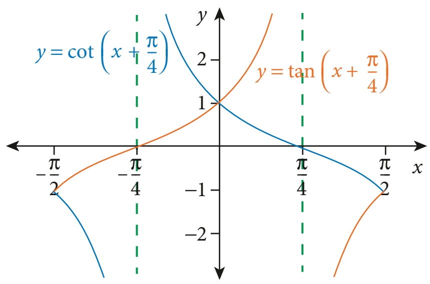

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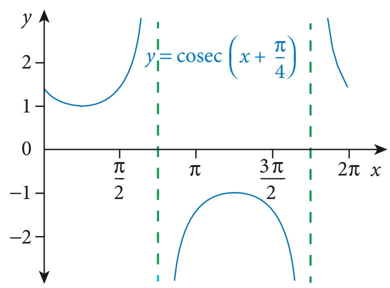

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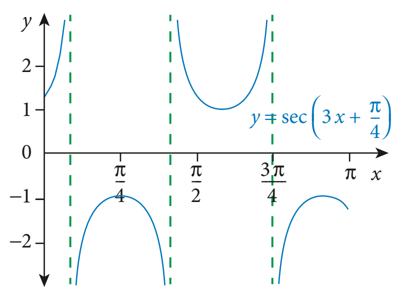

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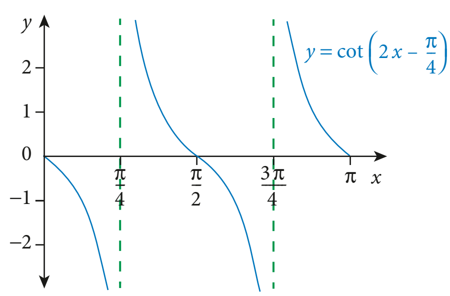

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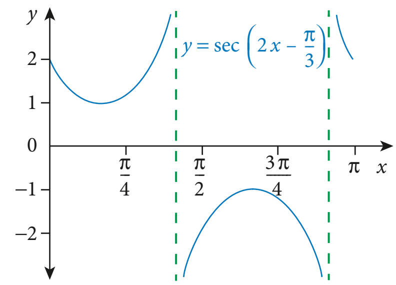


12.06

1 a $\sqrt{5}\sin(\theta + 26°34')$ b $2\sin(\theta + 60°)$

c $\sqrt{2}\sin(\theta + 45°)$ d $\sqrt{29}\sin(\theta + 21°48')$

e $\sqrt{17}\sin(\theta + 14°2')$ f $\sqrt{10}\sin(\theta + 18°26')$

g $\sqrt{13}\sin(\theta + 56°19')$ h $\sqrt{65}\sin(\theta + 60°15')$

i $\sqrt{41}\sin(\theta + 38°40')$ j $\sqrt{34}\sin(\theta + 59°2')$

2 a $\sqrt{2}\sin(\theta - 45°)$ b $\sqrt{5}\sin(\theta - 63°26')$

c $2\sin(\theta - 60°)$ d $2\sin(\theta - 30°)$

e $\sqrt{29}\sin(\theta - 21°48')$

3 $\sqrt{10}\cos(\theta - 18°26')$

4 $2\cos(\theta + 60°)$

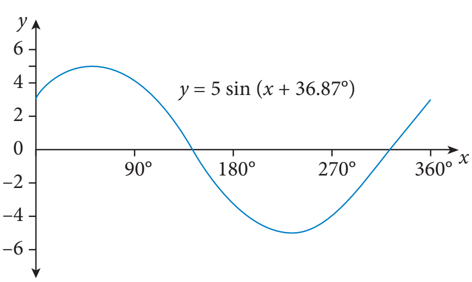
5 a $\sqrt{85}\sin(\theta + 12°28')$ b $\sqrt{85}\cos(\theta - 77°28')$

12.07

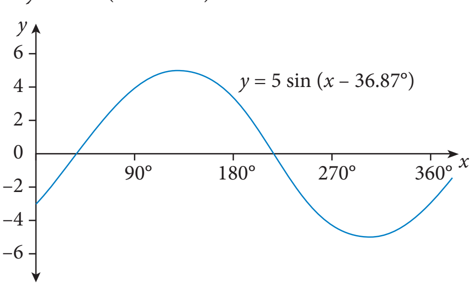
1 $r = 5$, $5\sin(x + \alpha) = 5[\sin(x)\cos(\alpha) + \cos(x)\sin(\alpha)] = 5\left[\sin(x) \times \frac{4}{5} + \cos(x) \times \frac{3}{5}\right]$

Where $\tan(\alpha) = \frac{3}{4}$, $\therefore \alpha = 36.87°$.

$y = 5\sin(x + 36.87°)$



2 $y = 5\sin(x - 36.87°)$



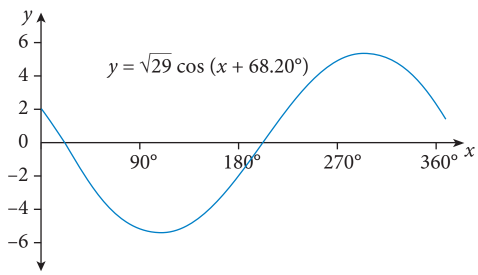
9780170250276

3 $r = \sqrt{29}$, $\sqrt{29}\ \cos(x + \alpha) = \sqrt{29}\ [\cos(x)\cos(\alpha) - \sin(x)\sin(\alpha)]$

$= \sqrt{29}\left[\cos(x) \times \frac{2}{\sqrt{29}} - \sin(x) \times \frac{5}{\sqrt{29}}\right]$

Where $\tan(\alpha) = \frac{5}{2}$, $\therefore\ \alpha = 68.20°$.

$y = \sqrt{29}\ \cos(x + 68.20°)$



4 a $\theta = 126°52', 306°52'$
b $\theta = 35°58', 189°16'$
c $\theta = 60°, 240°$
d $\theta = 180°, 270°$
e $\theta = 240°43', 327°21'$
f $\theta = 90°, 180°$
g $\theta = 90°, 340°32'$
h $\theta = 56°34', 176°34'$
i $\theta = 51°2', 190°5'$
j $\theta = 160°32', 270°$

5 $\theta = 2\pi k + 1.451,\ \theta = 2\pi k + 3.545,\ k \in \boldsymbol{Z}$

6 $\theta = 2\pi k,\ \theta = 2\pi k + \frac{2\pi}{3},\ k \in \boldsymbol{Z}$

7 a $h = 9\sin\left(\frac{\pi t}{4}\right) + \cos\left(\frac{\pi t}{4}\right)$

$\tan(\alpha) = \frac{1}{9}$, $\cos(\alpha) = \frac{9}{\sqrt{82}}$, $\sin(\alpha) = \frac{1}{\sqrt{82}}$, $r = \sqrt{82}$,

$\tan(\alpha) = \frac{1}{9}$, $\therefore\ \alpha = 0.1107^c$

Using $r\sin\left(\frac{\pi t}{4} + \alpha\right) =$

$r\left[\sin\left(\frac{\pi t}{4}\right)\cos(\alpha) + \cos\left(\frac{\pi t}{4}\right)\sin(\alpha)\right]$

gives

$\sqrt{82}\sin\left(\frac{\pi t}{4} + 0.1107\right) = \sqrt{82}\sin\left(\frac{\pi t}{4}\right) \times \frac{9}{\sqrt{82}} + \cos\left(\frac{\pi t}{4}\right) \times \frac{1}{\sqrt{82}}$

This gives $h = 9\sin\left(\frac{\pi t}{4}\right) + \cos\left(\frac{\pi t}{4}\right)$

$= \sqrt{82}\sin\left(\frac{\pi t}{4} + 0.1107\right)$

b 1 metre

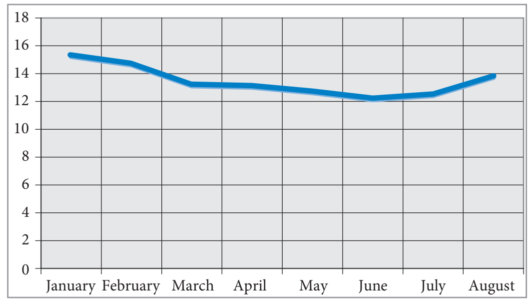
c $h = \sqrt{82}\sin\left(\frac{\pi t}{4} + 0.1107\right)$ so at $t = 2$, $h = 9$ metres.

d Solutions $t = 0$, $t = 3.718$, $t = 8$, $t = 11.718$, $t = 16$, $t = 19.718$, $t = 24$; therefore: returns at 12:43 p.m., 5 p.m., 8:43 p.m., 1 a.m. (next day), 4:43 a.m. (next day), 9 a.m. (next day).

12.08

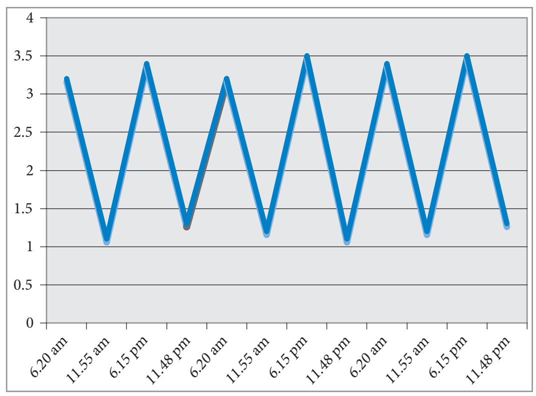
1 a 1300
b i 1600 ii 1010
c Amplitude 300, period 10 years

2 a



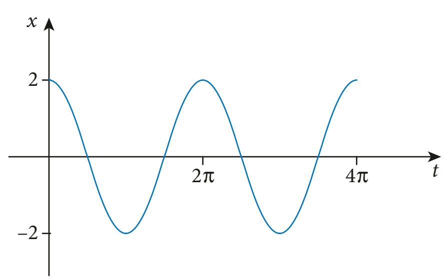
b It may be periodic – hard to tell from this data. Period would be about 10 months.
c Amplitude is 1.5 if it is periodic.

3 a

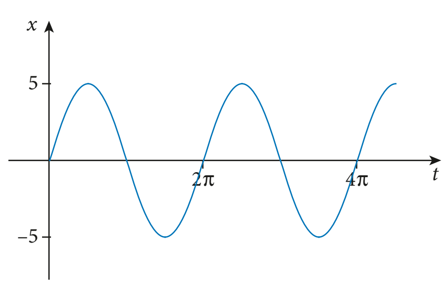


b Period 12 hours, amplitude 1.25. c 2.5 m

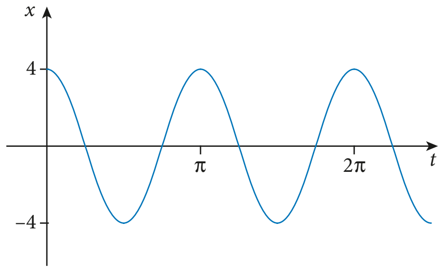
4



5



6 a $x = 4\cos(2t)$



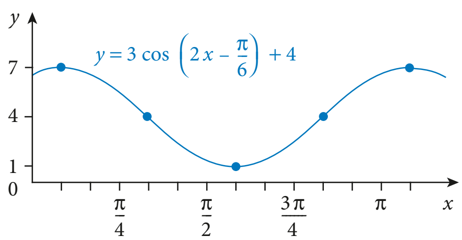
b $t = 0, \pi, 2\pi, \ldots, x = 4$

7 a $t = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \ldots$ b 2π c $x = 2$

CHAPTER 12 REVIEW

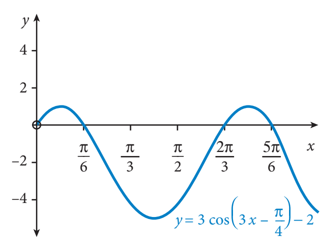
1 B
2 D
3 D
4 C
5 D

6 a $\left(\frac{\pi}{12}, 7\right), \left(\frac{\pi}{3}, 4\right), \left(\frac{7\pi}{12}, 1\right), \left(\frac{5\pi}{6}, 4\right), \left(\frac{13\pi}{12}, 7\right)$

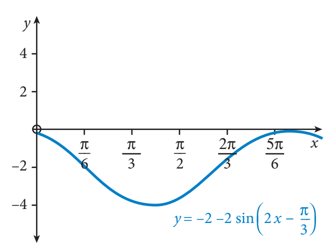


b

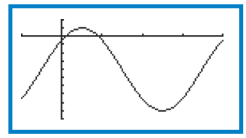
$\left(\frac{\pi}{12}, 1\right), \left(\frac{5\pi}{12}, -5\right), \left(\frac{3\pi}{4}, 1\right), \left(\frac{5\pi}{12}, -2\right), \left(\frac{7\pi}{12}, -1\right)$



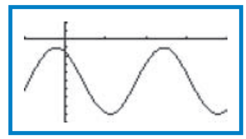
c $(0, -0.27), \left(\frac{5\pi}{12}, -4\right), \left(\frac{11\pi}{12}, 0\right), (\pi, -0.27)$



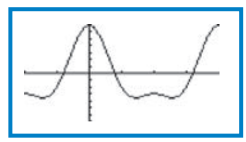
7 a $y = 5\sin\left(2x + \frac{\pi}{4}\right) - 4$



b $y = 4\cos\left(3x + \frac{\pi}{6}\right) - 5$



c $y = 4\cos(x) + 2\cos(2x) + \cos(4x)$



8 $x = 0, x = 0.948$

9 $x = 0.785, x = 3.927$

10 $\frac{\pi}{4}, \frac{7\pi}{12}, \frac{5\pi}{4}, \frac{19\pi}{12}$

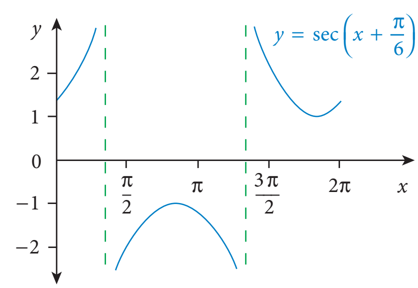
11 a $\alpha = \pi, 2\pi$ b $\alpha = \frac{5\pi}{3}$ c $\alpha = \frac{5\pi}{4}$

d $\alpha = \pi, 2\pi$ e $\alpha = \frac{11\pi}{6}$ f $\alpha = \frac{11\pi}{6}$

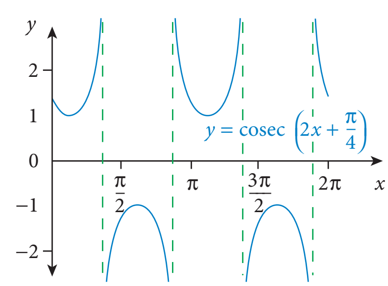
g $\alpha = \frac{5\pi}{4}$ h $\alpha = \frac{4\pi}{3}$ i $\alpha = \frac{15\pi}{8}$

j No solution in the domain

12 a



b



13 **a** $\frac{\sqrt{3}}{3}$ **b** $\sqrt{2}$ **c** $\frac{2\sqrt{3}}{3}$ **d** $-\frac{2\sqrt{3}}{3}$

14 $4\cos(x) + 3\sin(x) = 5\cos(x - \alpha)$, where $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$.

15 $6\cos(x) + \sin(x) = \sqrt{37}\cos(x - \alpha)$, where $\alpha = \tan^{-1}\left(\frac{1}{6}\right)$. $x = 90°$, $x = 288.92°$.

16 **a** Maximum = 12 m, minimum = 2 m
b 120
c $a = 5$, $b = 4\pi$, $c = 7$
d $33\frac{1}{3}\%$

17 **a** $a = 200$, $b = 50$, $m = 2\pi$, $g(t) = 200 + 50\sin(2\pi t)$
b $c = 200$, $d = 50$, $n = 4\pi$, $b(t) = 200 - 50\sin(4\pi t)$
c

***t* (years)**	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	**1**
Population	243 (240)	200	157 (160)	200

***t* (years)**	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	2
Population	243 (240)	200	157 (160)	200

d 12 days

MIXED REVISION 4

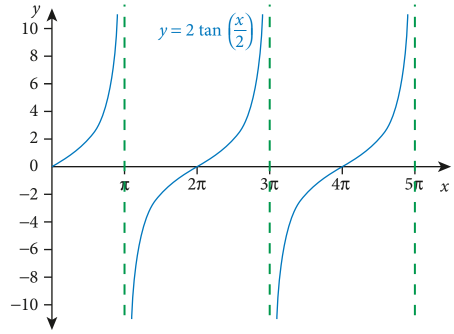
Multiple choice

1 B
2 D
3 D
4 C
5 D
6 B
7 A
8 D
9 E

Short answer

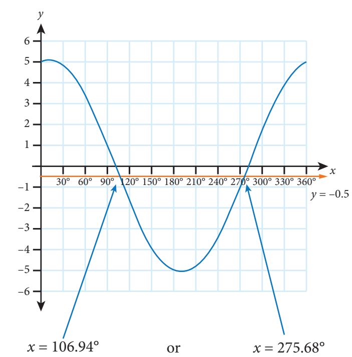
1 $(x - 2 - i)(x - 2 + i)$
2 $A'(-6, 2)$, $B'(6, 8)$, $C'(21, 4)$, $D'(12, -6)$
3 $\sqrt{13}\cos(\theta - \alpha)$ where $\alpha = \tan^{-1}\left(\frac{2}{3}\right) \approx 33.69$
4 **a** $3 - 7i$ **b** $\sqrt{37}$
5 $A'\left(-\frac{2+\sqrt{3}}{2}, \frac{1-2\sqrt{3}}{2}\right)$,
$B'\left(\frac{3+2\sqrt{3}}{2}, \frac{3\sqrt{3}-2}{2}\right)$,
$C'\left(\frac{4+7\sqrt{3}}{2}, \frac{4\sqrt{3}-7}{2}\right)$

6



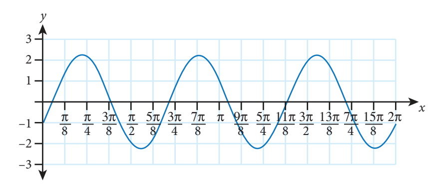
Application

1 **a** $\overrightarrow{AD} = v - z$, $\overrightarrow{AB} = w - z$ **b** $u = v + w - z$
2 $u = -1 + 2i$
3 A reflection in the line through the origin with inclination 40°.
4 Proof
5 **a** $\sqrt{26}\sin(x + \alpha)$, where $\theta = \tan^{-1}(5) = 78.69°$, equals $\sqrt{26}\sin(x + 78.69°)$.
b $x = n \times 360° - 84.32°$ or $x = n \times 360° + 106.94°$ where $n \in \mathbb{Z}$ (other answers are acceptable).
c



Solutions: $x = 106.94°$, $x = 275.68°$

6 **a**



Graph oscillates in symmetry about the average position and the motion can be expressed in the form of SHM using
$a\sin(x) - b\cos(x) = r\sin(x - \alpha)$
b amplitude = $\sqrt{5}$, period = $\frac{2\pi}{3}$
c 2.236 metres either side of the origin.